Section 6.2 Exercises

1. $\int (\cos x - 3x^2)\,dx = \sin x - x^3 + C$

3. $\int \left(t^2 - \frac{1}{t^2}\right) dt = \frac{t^3}{3} + t^{-1} + C$

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\,dx = \frac{3}{5}x^5 + x^{-2} + \tan x + C$

17. $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\int \sin 3x\,dx = \frac{1}{3}\int \sin u\,du$$

$$= -\frac{1}{3}\cos u + C$$

$$= -\frac{1}{3}\cos 3x + C$$

Check: $\frac{d}{dx}\left(-\frac{1}{3}\cos 3x + C\right) = \frac{1}{3}(-\sin 3x)(3) = \sin 3x$

19. $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec u \tan u\,du$$

$$= \frac{1}{2}\sec u + C$$

$$= \frac{1}{2}\sec 2x + C$$

21. $u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3\,du = dx$

$$\int \frac{dx}{x^2 + 9} = \int \frac{3du}{9u^2 + 9}$$

$$= \frac{3}{9}\int \frac{du}{u^2 + 1}$$

$$= \frac{1}{3}\int \frac{du}{u^2 + 1}$$

$$= \frac{1}{3}\tan^{-1} u + C$$

25. Let $u = 1 - x$

$du = -dx$

$$\int \frac{dx}{(1 - x^2)} = -\int \frac{du}{u^2}$$

$$= u^{-1} + C$$

$$= \frac{1}{1 - x} + C$$

27. Let $u = \tan x$

$du = \sec^2 x\,dx$

$$\int \sqrt{\tan x}\sec^2 x\,dx = \int u^{1/2}du$$

$$= \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}(\tan x)^{3/2} + C$$

31. Let $u = 3z + 4$

$du = 3\,dz$

$\frac{1}{3}du = dz$

$$\int \cos(3z + 4)dz = \frac{1}{3}\int \cos u\,du$$

$$= \frac{1}{3}\sin u + C$$

$$= \frac{1}{3}\sin(3z + 4) + C$$

33. Let $u = \ln x$

$du = \frac{1}{x}dx$

$$\int \frac{\ln^6 x}{x}dx = \int u^6 du$$

$$= \frac{1}{7}u^7 + C$$

$$= \frac{1}{7}(\ln^7 x) + C$$

6.2 Day 2

37. Let $u = \cos(2t+1)$

$du = -\sin(2t+1)(2)dt$

$-\frac{1}{2}du = \sin(2t+1)dt$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt = -\frac{1}{2}\int u^{-2}du$$
$$= \frac{1}{2}u^{-1} + C$$
$$= \frac{1}{2\cos(2t+1)} + C$$
$$= \frac{1}{2}\sec(2t+1) + C$$

39. $\int \frac{dx}{x \ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$x\ du = dx$

$\int \frac{du}{u} = \ln u = \ln(\ln x) + C$

41. $\int \frac{x\ dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\ dx$

$\frac{du}{2x} = dx$

$\frac{1}{2}\int \frac{du}{x^2+1} = \frac{1}{2}\ln u + C$

$= \frac{1}{2}\ln(x^2+1) + C$

53. Let $u = y+1$

$du = dy$

$$\int_0^3 \sqrt{y+1}\ dy = \int_1^4 u^{1/2}du$$
$$= \frac{2}{3}u^{3/2}\Big]_1^4$$
$$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$$
$$= \frac{2}{8}(8) - \frac{2}{3} = \frac{14}{3}$$

55. Let $u = \tan x$

$du = \sec^2 x\,dx$

$$\int_{-\pi/4}^0 \tan x \sec^2 x\,dx = \int_{-1}^0 u\ du$$
$$= \frac{1}{2}u^2\Big]_{-1}^0$$
$$= \frac{1}{2}(0) - \frac{1}{2}(-1)^2$$
$$= -\frac{1}{2}$$

59. Let $u = t^5 + 2t$

$du = (5t^4 + 2)\,dt$

$$\int_0^1 \sqrt{t^5+2t}\ (5t^4+2)\,dt = \int_0^3 u^{1/2}\,du$$
$$= \frac{2}{3}u^{3/2}\Big]_0^3$$
$$= \frac{2}{3}(3)^{3/2}$$
$$= \frac{2}{3}\sqrt{27} = 2\sqrt{3}$$

61. $\int_0^7 \frac{dx}{x+2}$

$u = x+2$

$du = dx$

$\int_0^7 \frac{du}{u} = \ln u\Big|_0^7 = \ln(x+2)\Big|_0^7 = \ln\left(\frac{9}{2}\right) = 1.504$

63. $\int_1^2 \frac{dt}{t-3}$

$u = t-3$

$du = dt$

$\int_1^2 \frac{du}{u} = \ln u\Big|_1^2 = \ln(t-3)\Big|_1^2 = \ln\left(\frac{1}{2}\right) = -0.693$

65. $\int_{-1}^3 \frac{x\,dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}\int_{-1}^3 \frac{du}{u} = \frac{1}{2}\ln u\Big|_{-1}^3 = \frac{1}{2}\ln(x^2+1)\Big|_{-1}^3 = \frac{1}{2}\ln(5) = 0.805$