Old Name: Key

AP Calculus AB
Integration By Parts
$\int udv = uv - \int vdu$

1) $\int xe^{-x}dx$

$u = x \quad | \quad dv = e^{-x}dx$
$du = dx \quad | \quad v = -e^{-x}$

$= -xe^{-x} - \int -e^{-x}dx$
$= \boxed{-xe^{-x} - e^{-x} + C}$

2) $\int xe^{3x}dx$

$u = 3x$
$du = 3dx$
$\frac{1}{3}du = dx$

$u = x \quad | \quad dv = e^{3x}dx$
$du = dx \quad | \quad v = \frac{1}{3}e^{3x}$
$\frac{1}{3}\int e^u du$

$= x \cdot \frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}dx$
$= \frac{x}{3}e^{3x} - \frac{1}{3}\cdot\frac{1}{3}e^{3x} + C$
$= \boxed{\frac{xe^{3x}}{3} - \frac{1}{9}e^{3x} + C}$

3) $\int x^2e^x dx$

$u = x^2 \quad | \quad dv = e^x dx$
$du = 2xdx \quad | \quad v = e^x$

$= x^2e^x - \int e^x \cdot 2x\,dx$

$u = 2x \quad | \quad dv = e^x dx$
$du = 2dx \quad | \quad v = e^x$

$= x^2e^x - (2xe^x - \int e^x \cdot 2dx)$
$= \boxed{x^2e^x - 2xe^x + 2e^x + C}$

4) $\int x^2e^{-2x}dx$

$u = x^2 \quad | \quad dv = e^{-2x}dx$
$du = 2xdx \quad | \quad v = -\frac{1}{2}e^{-2x}$

$u = -2x$
$du = -2dx$
$-\frac{1}{2}du = dx$
$-\frac{1}{2}\int e^u du$
$-\frac{1}{2}e^{-2x}$

$= x^2 \cdot -\frac{1}{2}e^{-2x} + \frac{1}{2}\int e^{-2x} \cdot 2x\,dx$
$= -\frac{x^2}{2}e^{-2x} + \int e^{-2x} \cdot x\,dx$

$u = x \quad | \quad dv = e^{-2x}dx$
$du = dx \quad | \quad v = -\frac{1}{2}e^{-2x}$

$= -\frac{x^2}{2}e^{-2x} + x \cdot -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}dx$
$= \boxed{-\frac{x^2}{2}e^{-2x} - \frac{x}{2}e^{-2x} + -\frac{1}{4}e^{-2x} + C}$

5) $\int x \sin 2x dx$

$u = x \quad dv = \sin 2x\,dx$
$du = dx \quad v = -\frac{1}{2}\cos(2x)$

$= -\frac{x}{2}\cos(2x) - \int -\frac{1}{2}\cos 2x\,dx$

$u = 2x$
$du = 2dx$
$\frac{1}{2}\int \sin u\,du$
$-\frac{1}{2}\cos(2x)$

$= -\frac{x}{2}\cos(2x) + \frac{1}{2}\int \cos(2x)dx$

$u = 2x$
$du = 2dx$
$-\frac{1}{2}du = dx$
$\frac{1}{2}\int \cos u\,du$
$\frac{1}{2}\cdot\sin(2x)$

$= -\frac{x}{2}\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$
$= \boxed{-\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C}$

6) ~~$\int (\ln x)^2 dx$~~

6) $\int x\cos(3x)\,dx$

$u = 3x$
$du = 3dx$
$\frac{1}{3}du = dx$
$\frac{1}{3}\int \cos u$

$u = x \quad dv = \cos(3x)dx$
$du = dx \quad v = \frac{1}{3}\sin(3x)$

$= x \cdot \frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x)\,dx$
$= \boxed{\frac{x}{3}\sin(3x) + \frac{1}{9}\cos(3x) + C}$

$u = 3x$
$du = 3dx$
$\frac{1}{3}du = dx$
$\frac{1}{9}\int \sin(u)$
$\frac{1}{9}\cos 3x$

7) $\int x^2 \cos x dx$

$u = x^2$ | $dv = \cos x\, dx$
$du = 2x\,dx$ | $v = \sin x$

$= x^2 \sin x - \int \sin x \cdot 2x\, dx$

$u = 2x$ | $dv = \sin x dx$
$du = 2dx$ | $v = -\cos x$

$= x^2 \sin x - (-\cos x(2x) - \int -\cos x \cdot 2dx)$

$= \boxed{x^2 \sin x + 2x\cos x - 2\sin x + C}$

8) $\int x^2 \sin x dx$

$u = x^2$ | $dv = \sin x\, dx$
$du = 2x\,dx$ | $v = -\cos x$

$= -x^2 \cos x + \int \cos x \cdot 2x\, dx$

$u = 2x$ | $dv = \cos x\, dx$
$du = 2dx$ | $v = \sin x$

$= -x^2 \cos x + 2x \sin x - 2\int \sin x dx$

$= \boxed{-x^2 \cos x + 2x \sin x + 2\cos x + C}$

9) $\int \sqrt{x} \ln x dx$

$u = \ln x$ | $dv = \sqrt{x}\, dx$
$du = \frac{1}{x} dx$ | $v = x^{3/2} \cdot \frac{2}{3}$

$= \ln x \cdot \frac{2}{3} x^{3/2} - \int \frac{2}{3} x^{3/2} \cdot \frac{1}{x} dx$

$= \frac{2}{3} x^{3/2} \cdot \ln x - \int \frac{2}{3} x^{1/2} dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \cdot x^{3/2} \cdot \frac{2}{3} + C$

$= \boxed{\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C}$

10) $\int x \ln x dx$

11) $\int (\ln x)^2 dx$

$u = (\ln x)^2$ | $dv = dx$
$du = 2(\ln x) \cdot \frac{1}{x} \cdot dx$ | $v = x$

$= x(\ln x)^2 - \int x \cdot 2 \cdot \ln x \cdot \frac{1}{x} \cdot dx$

$= x(\ln x)^2 - 2\int \ln x\, dx$

$u = \ln x$ | $dv = dx$
$du = \frac{1}{x} dx$ | $v = x$

$= x(\ln x)^2 - 2(x \ln x - \int x \cdot \frac{1}{x} dx)$

$= \boxed{x(\ln x)^2 - 2x \ln x + 2x + C}$

12) $\int \frac{\ln x}{\sqrt{x}} dx$

13) $\int \ln(2x+3) dx$

14) $\int \ln(x^2+4) dx$

15) $\int \sin^{-1} x dx$