U - Substitution = Reverse Chain Rule 6.3 day 1

Integration by Parts = Reverse Product Rule

$\boxed{\text{Product Rule}}$ $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\int u \frac{dv}{\cancel{dx}} \cdot \cancel{dx} = \int \frac{d}{\cancel{dx}}(uv)\cancel{dx} - \int v \cdot \frac{du}{\cancel{dx}} \cdot \cancel{dx}$$

$$\int u\,dv = uv - \int v\,du$$

$$uv - \int v\,du = \int u\,dv$$

$\int x \cos x \, dx$

$u = x$

$\frac{du}{dx} = 1$

$du = dx$

$dv = \cos x \, dx$

$v = \sin x$

$\int u \, dv = uv - \int v \, du$

$\int x \cos x \, dx = x \cdot \sin x - \int \sin x \, dx$

$\boxed{= x \sin x + \cos x + C}$

$\int x\, e^x\, dx$

$u = x$ | $dv = e^x\, dx$
$du = dx$ | $v = e^x$

$\int u\, dv = uv - \int v\, du$

$\int x e^x dx = x \cdot e^x - \int e^x dx$

$= \boxed{xe^x - e^x + C}$

$vu' + uv'$ $\boxed{\text{Check}}$

$y = \underset{u\,v}{xe^x} - e^x$

$y' = e^x \cdot 1 + x \cdot e^x - e^x$

$y' = xe^x$

$\int 2t \cos 3t \, dt$

$u = 2t$
$du = 2dt$

$dv = \cos(3t)dt$
$v = \frac{1}{3}\sin(3t)$

$\int \cos(3t)dt$
$u = 3t$
$du = 3dt$
$\frac{1}{3}du = dt$
$\frac{1}{3}\int \cos u \, du$
$\frac{1}{3}\sin(3t)$

$\int u\,dv = uv - \int v\,du$

$\int 2t\cos(3t) = 2t \cdot \frac{1}{3}\sin(3t) - \int \frac{1}{3}\sin(3t)\cdot 2dt$

$= \frac{2t}{3}\sin(3t) - \frac{2}{3}\int \sin(3t)\,dt$

$u = 3t$
$du = 3dt$
$\frac{1}{3}du = dt$
$\frac{1}{3}\int \sin u \, du$
$-\frac{1}{3}\cdot\cos(3t)$

$= \frac{2}{3}t\sin(3t) - \frac{2}{3}\cdot\frac{-1}{3}\cos(3t)$

$= \frac{2}{3}t\sin(3t) + \frac{2}{9}\cos(3t) + C$

$\int x \ln x \, dx$

$u = \ln x$ | $dv = x\,dx$

$du = \frac{1}{x}\,dx$ | $v = \frac{x^2}{2}$

$= uv - \int v\,du$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x}{2}\,dx$

$= \ln x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + C$

$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$

Extra Example

$\int \ln x\,dx$

$u = \ln x$ $\quad dv = 1\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = x$

$uv - \int v \cdot du$

$= x \ln x - \int x \cdot \frac{1}{x}\,dx$

$= x \ln x - \int 1\,dx$

$= \boxed{x \ln x - x + C}$

6.3 day 1

1, 3, 5, 7, 9, 11, 15