5.4 Day 2

$\int x^n = \frac{x^{n+1}}{n+1}$

Ex 1) $\int_1^4 (x^2+x)dx = \left.\frac{x^3}{3}+\frac{x^2}{2}\right|_1^4$

$= \left(\frac{4^3}{3}+\frac{4^2}{2}\right) - \left(\frac{1^3}{3}+\frac{1^2}{2}\right)$

$= \frac{64}{3}+\frac{16}{2}-\frac{1}{3}-\frac{1}{2}$

$= \boxed{28.5}$

Ex 2) $\int_3^1 \left(\sqrt{(x)} + 1/x\right)dx$

$x^{1/2}$

$\left.\frac{x^{3/2}}{3/2}+\ln x\right|_3^1 = \frac{2\cdot 1^{3/2}}{3}+\ln 1-\left(3^{3/2}\cdot\frac{2}{3}+\ln 3\right)$

$\sqrt{27}=\sqrt{9}\sqrt{3}$

$= \frac{2}{3}+0-\sqrt{3^3}\cdot\frac{2}{3}-\ln 3$

$= \frac{2}{3}-2\sqrt{3}-\ln 3$

$\boxed{\approx -3.896}$

Ex 3) $\int_0^{\pi}(\cos x - \sin x)\,dx$

$$\sin x + \cos x \Big|_0^{\pi} = \sin\pi + \cos\pi - (\sin 0 + \cos 0)$$
$$= 0 + -1 - 0 - 1 = -2$$

Ex 4) $\int_{\pi}^{2\pi} 2(\sec^2 x)\,dx$

$\int \sec^2 x = \tan x$

$$2\int_{\pi}^{2\pi} \sec^2 x \cdot dx$$

$\frac{y}{x} = \frac{0}{1}$ $\quad \frac{y}{x} = \frac{0}{-1}$

$$2\left[\tan x \Big|_{\pi}^{2\pi}\right] = 2(\tan 2\pi - \tan\pi)$$
$$= 2(0 - 0) = 0$$

Ex 5) $\int_{-2}^{-1} 2/(x^2)\,dx = \int_{-2}^{-1} 2x^{-2} \cdot dx$

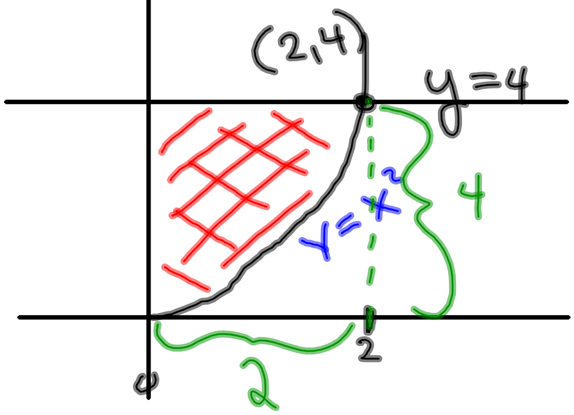
$$\frac{2x^{-1}}{-1} = -2x^{-1}\Big|_{-2}^{-1} = -2(-1)^{-1} - -2(-2)^{-1}$$
$$= \frac{-2}{-1} + \frac{2}{-2} = 2 - 1 = 1$$

Ex 6) $\int_{-1}^{1}(3x^2 - 4x + 7)\,dx$

$$= x^3 - 2x^2 + 7x \Big|_{-1}^{1} = 1^3 - 2(1)^2 + 7(1) - ((-1)^3 - 2(-1)^2 + 7(-1))$$
$$= 1 - 2 + 7 + 1 + 2 + 7$$
$$= 16$$

$$\frac{\cancel{3}x^3}{\cancel{3}} - 4 \cdot \frac{x^2}{2} + \frac{7x^{0+1}}{1}$$

Ex 7)



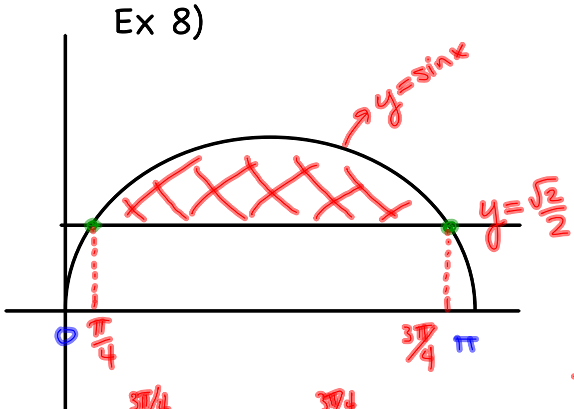
$$= \int_0^2 4\cdot dx - \int_0^2 x^2\cdot dx$$

$$= 8 - \left.\frac{x^3}{3}\right|_0^2$$

$$= 8 - \left(\frac{8}{3} - 0\right)$$

$$= \frac{3\cdot 8}{3} - \frac{8}{3} = \frac{16}{3}$$

Ex 8)



$\sin x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$

$x = \frac{3\pi}{4}$

$$\int_{\pi/4}^{3\pi/4} \sin x\cdot dx - \int_{\pi/4}^{3\pi/4} \frac{\sqrt{2}}{2} dx = -\cos x\Big|_{\pi/4}^{3\pi/4} - \frac{\sqrt{2}}{2}x\Big|_{\pi/4}^{3\pi/4}$$

$$= -\cos\frac{3\pi}{4} + \cos\frac{\pi}{4} - \frac{\sqrt{2}}{2}\cdot\frac{3\pi}{4} + \frac{\sqrt{2}}{2}\cdot\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \frac{3\pi\sqrt{2}}{8} + \frac{\sqrt{2}\pi}{8} = \frac{2\sqrt{2}}{2} - \frac{2\pi}{8} = \sqrt{2} - \frac{\pi}{4}\sqrt{2}$$

$$= .303$$

* <u>Note</u>: Evaluating an integral gives you <u>net</u> area
(above x-axis is (+)
(below x-axis is (-))

<u>Total Area</u>
Add the Absolute values of the integrals.

<u>Calculator</u>
* fnInt(|f(x)|, x, a, b)