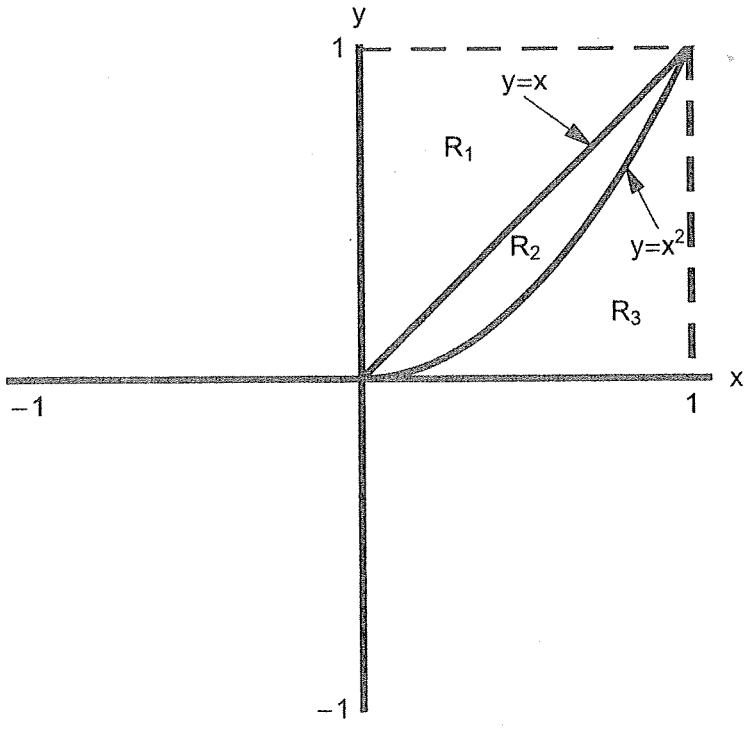
Key

AP Calculus AB
Worksheet: Volumes



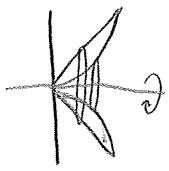
Let R_1 be the region in the first quadrant bounded by y = x, the y-axis, and the line y = 1. Let R_2 be the region in the first quadrant bounded by $y = x^2$ and y = x. Let R_3 be the region in the first quadrant bounded by $y = x^2$, the x-axis, and the line x = 1.

Write an integral expression for the volume of the solid described. Do not evaluate.

1. The volume of the solid generated by revolving R_3 about the x-axis.

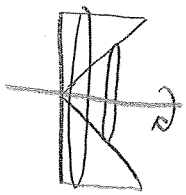
$$V = \pi \int_0^1 (x^2)^2 \, dx = \int_0^1 \pi x^4 \, dx$$

2. The volume of the solid generated by revolving R_2 about the x-axis.



$$V = \int_0^1 \pi(x)^2 - \pi (x^2)^2 \, dx = \int_0^1 (\pi x^2 - \pi x^4) \, dx$$

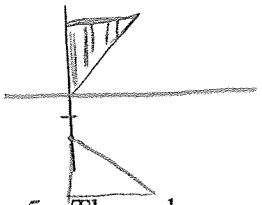
3. The volume of the solid generated by revolving R_1 about the x-axis.



$$V = \int_0^1 (\pi(1)^2 - \pi(x)^2) \, dx$$

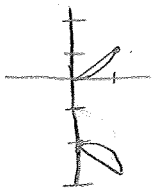
$$V = \int_0^1 (\pi - \pi x^2) \, dx$$

4. The volume of the solid generated by revolving R_1 about the line y = -1.



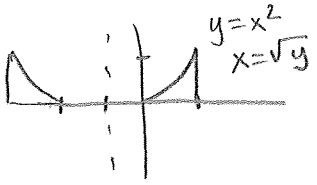
$$V = \int_0^1 \left(\pi(1+1)^2 - \pi(x+1)^2\right)dx = \int_0^1 \left(4\pi - \pi(x+1)^2\right)dx$$

5. The volume of the solid generated by revolving R_2 about the line y = -1.



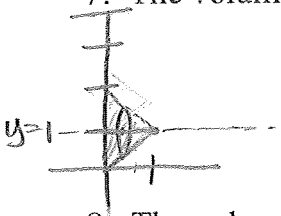
$$V = \int_0^1 \left(\pi(x+1)^2 - \pi(x^2+1)^2\right)dx$$

6. The volume of the solid generated by revolving R_3 about the line x = -1.



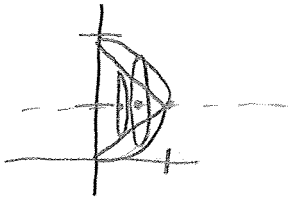
$$V = \int_0^1 \pi(2)^2 - \pi(\sqrt{y}+1)^2\,dx$$

7. The volume of the solid generated by revolving R_1 about the line y = 1.



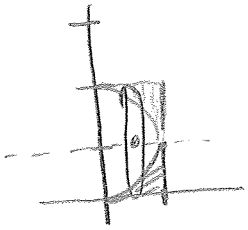
$$V = \int_0^1 \pi(1-x)^2\,dx$$

8. The volume of the solid generated by revolving R_2 about the line y = 1.



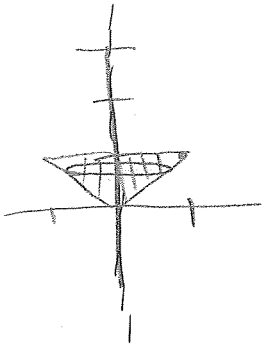
$$V = \int_0^1 \left(\pi(1-x^2)^2 - (1-x)^2\right)dx$$

9. The volume of the solid generated by revolving R_3 about the line y = 1.



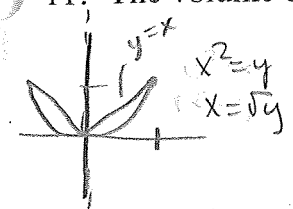
$$V = \int_0^1 \left(\pi(1)^2 - \pi(1-x^2)^2\right)dx$$

10. The volume of the solid generated by revolving R_1 about the y-axis.



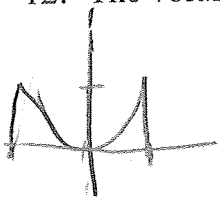
$$V = \pi\int_0^1 y^2\,dy = \int_0^1 \pi y^2\,dy$$

11. The volume of the solid generated by revolving R_2 about the y-axis.



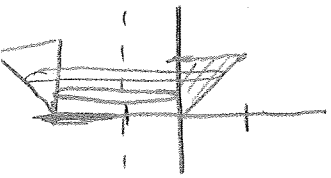
$$V = \int_0^1 \pi(\sqrt{y})^2 - \pi(y)^2 dy$$

12. The volume of the solid generated by revolving R_3 about the y-axis.



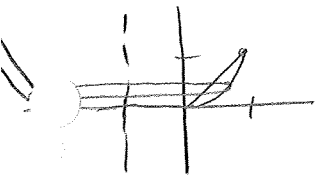
$$V = \int_0^1 (\pi(1)^2 - \pi(\sqrt{y})^2) dy$$

13. The volume of the solid generated by revolving R_1 about the line x = -1.



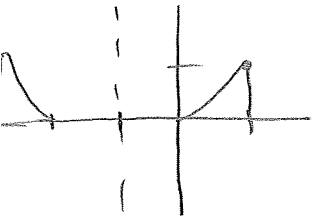
$$V = \int_0^1 \pi(y+1)^2 - \pi(1)^2) dy$$

14. The volume of the solid generated by revolving R_2 about the line x = -1.



$$V = \int_0^1 (\pi(\sqrt{y}+1)^2 - \pi(y+1)^2 dy$$

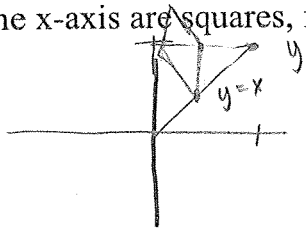
15. The volume of the solid generated by revolving R_3 about the line x = -1.



$$V = \int_0^1 (\pi(2)^2 - \pi(\sqrt{y}+1)^2) dy$$

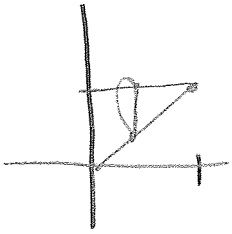
16. Let R_1 be the base of a solid in the x-y plane. If cross sections of the solid perpendicular to the x-axis are squares, find the volume of the solid.

$A = s^2 = (1-x)^2$



$$V = \int_0^1 (1-x)^2 dx$$

17. Let R_1 be the base of a solid in the x-y plane. If cross sections of the solid perpendicular to the x-axis are semicircles, find the volume of the solid.



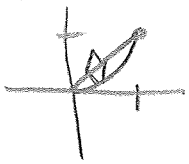
$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi\left(\frac{1-x}{2}\right)^2$

$$V = \frac{1}{2}\int_0^1 \pi\left(\frac{1-x}{2}\right)^2 dx$$

18. Let R_2 be the base of a solid in the x-y plane. If cross sections of the solid perpendicular to the x-axis are squares, find the volume of the solid.

$A = s^2 = (x - x^2)^2$

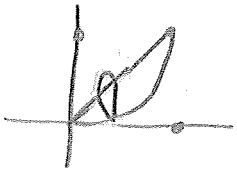


$$V = \int_0^1 (x - x^2)^2 \, dx$$

19. Let R_2 be the base of a solid in the x-y plane. If cross sections of the solid perpendicular to the x-axis are semicircles, find the volume of the solid.

$A = \frac{1}{2}\pi r^2$

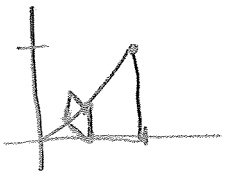
$= \frac{1}{2}\pi \left(\frac{x - x^2}{2}\right)^2$



$$V = \int_0^1 \left(\frac{1}{2}\pi \left(\frac{x - x^2}{2}\right)^2\right) dx$$

20. Let R_3 be the base of a solid in the x-y plane. If cross sections of the solid perpendicular to the x-axis are squares, find the volume of the solid.

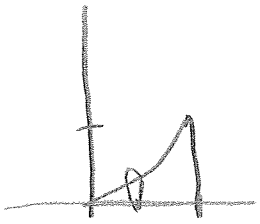
$A = s^2 = (x^2 - 0)^2$



$$V = \int_0^1 (x^2)^2 \, dx$$

21. Let R_3 be the base of a solid in the x-y plane. If cross sections of the solid perpendicular to the x-axis are semicircles, find the volume of the solid.

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(x^2/2\right)^2$



$$V = \int_0^1 \frac{1}{2}\pi \left(\frac{x^2}{2}\right)^2 dx$$