

Name Key

Quiz Section _____

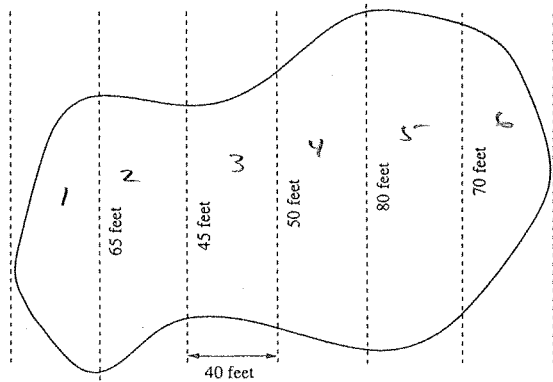
In this work sheet we'll study the problem of finding the area of a region bounded by curves. We'll first estimate an area given numerical information. Then we'll use calculus to find the area of a more complicated region.

The Lake

1 The widths, in feet, of a small lake were measured at 40 foot intervals. Estimate the area of the lake.

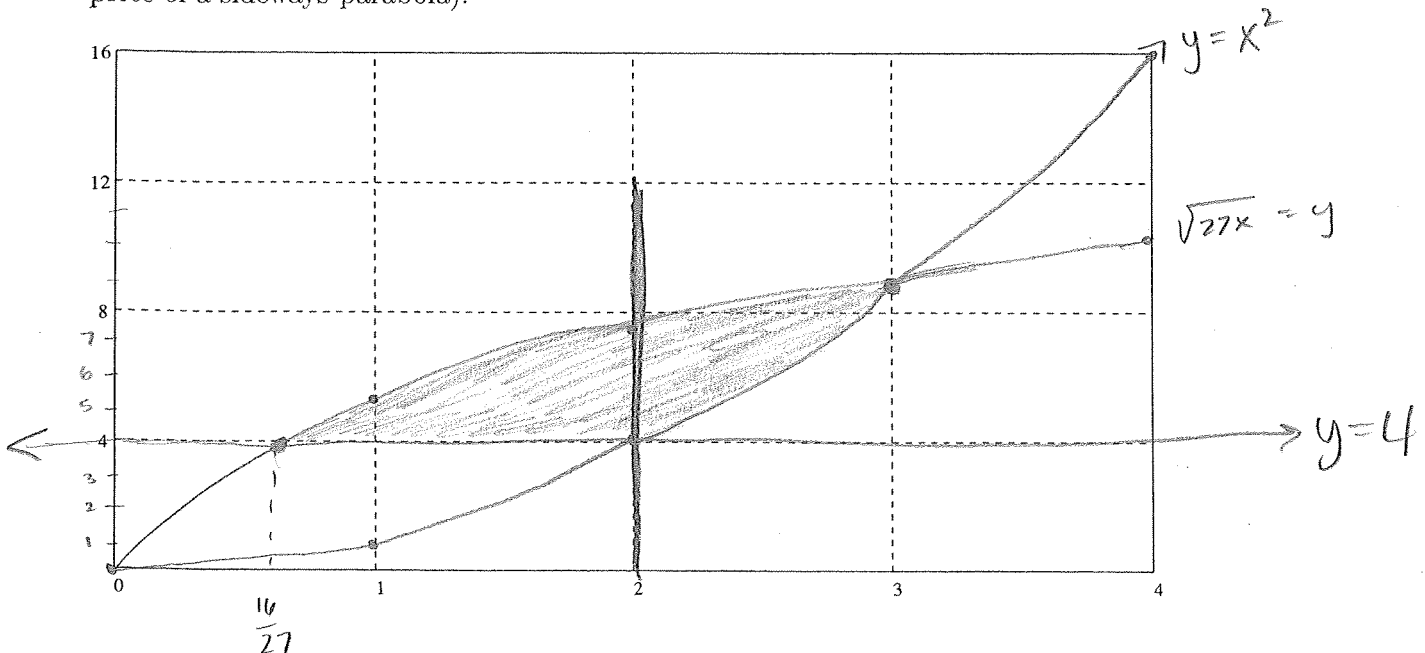
$$\begin{aligned} \#1 & \frac{1}{2} \cdot 40 (0 + 65) = \\ \#2 & \frac{1}{2} \cdot 40 (65 + 45) = \\ \#3 & \frac{1}{2} \cdot 40 (45 + 50) = \\ \#4 & \frac{1}{2} \cdot 40 (50 + 80) = \\ \#5 & \frac{1}{2} \cdot 40 (80 + 70) = \\ \#6 & \frac{1}{2} \cdot 40 (70 + 0) = \end{aligned}$$

$12,400 \text{ ft}^2$



Area Bounded by Three Curves

2 On the grid below sketch the graphs of $y = 4$, $y = x^2$ and $y = \sqrt{27x}$. (The last one is just a piece of a sideways parabola).



3 Shade the "triangular" region bounded by the graphs of the three functions that lies above the horizontal line.

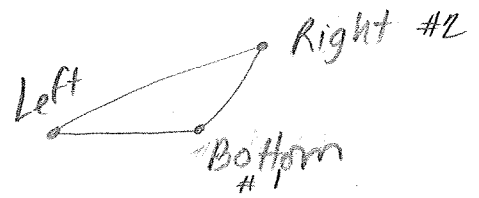
4 Compute the x -coordinate of the left endpoint of the region.

$$y = 4 \quad y = \sqrt{27x}$$

$$4 = \sqrt{27x}$$

$$16 = 27x$$

$$\boxed{x = \frac{16}{27}}$$



5 Compute the x -coordinate of the right endpoint of the region.

which one?

$$\begin{aligned} \#2 \\ \sqrt{27x} &= x^2 \\ 27x &= x^4 \end{aligned}$$

$$27 = x^3$$

$$\boxed{x = 3}$$

6 Note that the top of the region consists of a single curve, but the bottom of the region consists of two different curves. Find the x -coordinate where these two curves meet.

#1

$$4 = x^2$$

$$2 \pm = x$$

$$\boxed{x = 2}$$

7 Sketch in a vertical line at the x -coordinate you found in the last problem. This divides the region into two smaller sub-regions.

8 Compute the area of the left sub-region.

$$\int_{16/27}^2 (\sqrt{27x} - 4) dx$$

$$\begin{aligned} u &= 27x \\ du &= 27 dx \end{aligned}$$

$$\frac{1}{27} \int u^{1/2} - \int 4 dx = \frac{1}{27} (27x)^{3/2} \cdot \frac{2}{3} \Big|_{16/27}^2 - 4x \Big|_{16/27}^2$$

$$= \boxed{2.588}$$

9 Compute the area of the right sub-region. Add the two areas together to get the total area.

$$\int_2^3 (\sqrt{27x} - x^2) dx = 1.8687$$

$$\begin{array}{r} 1.8687 \\ + 2.588 \\ \hline \boxed{4.457} \end{array}$$