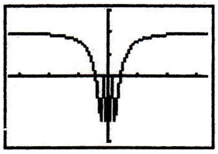
Section 2.2 Exercises

1.



[−5, 5] by [−1.5, 1.5]

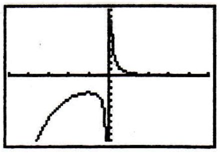
X	Y1
100	.99995
200	.99999
300	.99999
400	1
-100	.99995
-200	.99999
-300	.99999
Y1 = cos(1/X)	

(a) $\lim_{x\to\infty} f(x) = 1$

(b) $\lim_{x\to-\infty} f(x) = 1$

(c) $y = 1$

3.



[−5, 5] by [−10, 10]

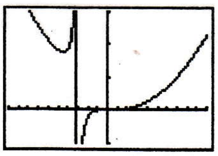
X	Y1
100	4E-46
200	7E-90
300	0
400	0
-100	-3E41
-200	-4E84
-300	ERROR
Y1 = e^(-X)/X	

(a) $\lim_{x\to\infty} f(x) = 0$

(b) $\lim_{x\to-\infty} f(x) = -\infty$

(c) $y = 0$

4.



[−10, 10] by [−100, 300]

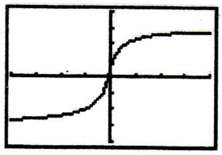
X	Y1
100	29125
200	118226
300	267326
400	476426
-100	30927
-200	121826
-300	272726
Y1 = (3X³-X+1)/(X...	

(a) $\lim_{x\to\infty} f(x) = \infty$

(b) $\lim_{x\to-\infty} f(x) = \infty$

(c) No horizontal asymptotes.

5.



[−20, 20] by [−4, 4]

X	Y1
100	2.951
200	2.9752
300	2.9834
400	2.9876
-100	-2.931
-200	-2.965
-300	-2.977
Y1 = (3X+1)/(abs(...	

(a) $\lim_{x\to\infty} f(x) = 3$

(b) $\lim_{x\to-\infty} f(x) = -3$

(c) $y = 3$, $y = -3$

9. $0 \le 1-\cos x \le 2$. So, for $x > 0$ we have $0 \le 1-\cos x \le \frac{1}{x^2}$.

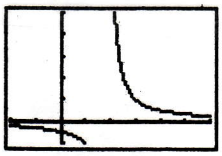
By the Sandwich Theorem,

$$0 = \lim_{x\to\infty}(0) = \lim_{x\to\infty}\frac{1-\cos x}{x^2} = \lim_{x\to\infty}\frac{1}{x^2} = 0.$$

11. $-1 \le \sin x \le 1$

$$\lim_{x\to-\infty}\frac{-1}{x} = \lim_{x\to-\infty}\frac{1}{x} = 0.$$

13.

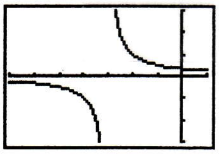


[−2, 6] by [−1, 5]

X	Y1
2.8	1.25
2.4	2.5
2.2	5
2.1	10
2.01	100
2.001	1000
2.0001	10000
Y1 = 1/(X-2)	

$$\lim_{x\to2^+}\frac{1}{x-2} = \infty$$

15.



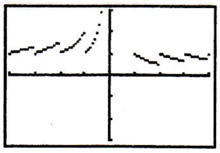
[−7, 1] by [−3, 3]

X	Y1
-3.8	-1.25
-3.4	-2.5
-3.3	-3.333
-3.2	-5
-3.1	-10
-3.01	-100
-3.001	-1000

Y1=1/(X+3)

$$\lim_{x\to -3^-} \frac{x}{x+3} = -\infty$$

17.



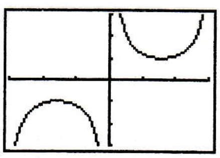
[−4, 4] by [−3, 3]

X	Y1
.8	0
.4	0
.2	0
.1	0
.01	0
.001	0
1E-4	0

Y1=int(X)/X

$$\lim_{x\to 0^+} \frac{\operatorname{int} x}{x} = 0$$

19.



[−3, 3] by [−3, 3]

X	Y1
.8	1.394
.4	2.5679
.2	5.0335
.1	10.017
.01	100
.001	1000
1E-4	10000

Y1=1/sin(X)

$$\lim_{x\to 0^+} \csc x = \infty$$

21. $y = \left(2 - \frac{x}{x+1}\right)\left(\frac{x^2}{5+x^2}\right) = \left(\frac{2(x+1)-x}{x+1}\right)\left(\frac{x^2}{5+x^2}\right)$

$= \left(\frac{x+2}{x+1}\right)\left(\frac{x^2}{5+x^2}\right) = \frac{x^3+2x^2}{x^3+x^2+5x+5}$

An end behavior model for y is $\frac{x^3}{x^3} = 1$.

$\lim_{x\to\infty} y = \lim_{x\to\infty} 1 = 1$

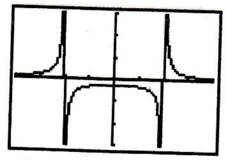
$\lim_{x\to -\infty} y = \lim_{x\to -\infty} 1 = 1$

23. Use the method of Example 10 in the text.

$$\lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}} = \lim_{x\to 0^+} \frac{\cos x}{1+x} = \frac{\cos(0)}{1+0} = \frac{1}{1} = 1$$

$$\lim_{x\to -\infty} \frac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}} = \lim_{x\to 0^-} \frac{\cos x}{1+x} = \frac{\cos(0)}{1+0} = \frac{1}{1} = 1$$

27.

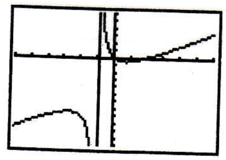


[−4, 4] by [−3, 3]

(a) $x = -2$, $x = 2$

(b) Left-hand limit at -2 is ∞.
Right-hand limit at -2 is $-\infty$.
Left-hand limit at 2 is $-\infty$.
Right-hand limit at 2 is ∞.

29.



[−6, 6] by [−12, 6]

(a) $x = -1$

(b) Left-hand limit at -1 is $-\infty$.
Right-hand limit at -1 is ∞.

35. An end behavior model is $\frac{2x^3}{x} = 2x^2$. (a)

36. An end behavior model is $\frac{x^5}{2x^2} = 0.5x^3$. (c)

37. An end behavior model is $\frac{2x^4}{-x} = -2x^3$. (d)

38. An end behavior model is $\frac{x^4}{-x^2} = -x^2$. (b)

41. (a) $\frac{x}{2x^2} = \frac{1}{2x}$

(b) $y = 0$

43. (a) $\frac{4x^3}{x} = 4x^2$

(b) None

45. (a) The function $y = e^x$ is a right end behavior model because $\lim_{x\to\infty} \frac{e^x - 2x}{e^x} = \lim_{x\to\infty}\left(1 - \frac{2x}{e^x}\right) = 1 - 0 = 1.$

(b) The function $y = -2x$ is a left end behavior model because $\lim_{x\to-\infty} \frac{e^x - 2x}{-2x} = \lim_{x\to-\infty}\left(-\frac{e^x}{2x} + 1\right) = 0 + 1 = 1.$

48. (a, b) The function $y = x^2$ is both a right end behavior model and a left end behavior model because

$$\lim_{x\to\pm\infty}\left(\frac{x^2 + \sin x}{x^2}\right) = \lim_{x\to\pm\infty}\left(1 + \frac{\sin x}{x^2}\right) = 1.$$