Section 2.3 Exercises

1. The function $y=\dfrac{1}{(x+2)^2}$ is continuous because it is a quotient of polynomials, which are continuous. Its only point of discontinuity occurs where it is undefined. There is an infinite discontinuity at $x=-2$.

3. The function $y=\dfrac{1}{x^2+1}$ is continuous because it is a quotient of polynomials, which are continuous. Furthermore, the domain is all real numbers because the denominator, x^2+1, is never zero. Since the function is continuous and has domain $(-\infty, \infty)$, there are no points of discontinuity.

5. The function $y=\sqrt{2x+3}$ is a composition $(f\circ g)(x)$ of the continuous functions $f(x)=\sqrt{x}$ and $g(x)=2x+3$, so it is continuous. Its points of discontinuity are the points not in the domain, i.e., all $x<-\dfrac{3}{2}$.

7. The function $y=\dfrac{|x|}{x}$ is equivalent to

$$y=\begin{cases}-1, & x<0\\ 1, & x>0.\end{cases}$$

It has a jump discontinuity at $x=0$.

9. The function $y=e^{1/x}$ is a composition $(f\circ g)(x)$ of the continuous functions $f(x)=e^x$ and $g(x)=\dfrac{1}{x}$, so it is continuous. Its only point of discontinuity occurs at $x=0$, where it is undefined. Since $\lim_{x\to 0^+} e^{1/x}=\infty$, this may be considered an infinite discontinuity.

11. (a) Yes, $f(-1)=0$.

(b) Yes, $\lim_{x\to -1^+} = 0$.

(c) Yes

(d) Yes, since -1 is a left endpoint of the domain of f and $\lim_{x\to -1^+} f(x)=f(-1)$, f is continuous at $x=-1$.

12. (a) Yes, $f(1)=1$.

(b) Yes, $\lim_{x\to 1} f(x)=2$.

(c) No

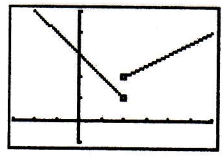
(d) No

13. (a) No

(b) No, since $x=2$ is not in the domain.

15. Since $\lim_{x\to 2} f(x)=0$, we should assign $f(2)=0$.

19.

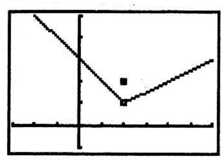


$[-3, 6]$ by $[-1, 5]$

(a) $x=2$

(b) Not removable, the one-sided limits are different.

20.

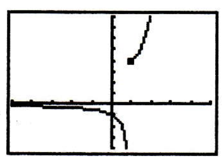


$[-3, 6]$ by $[-1, 5]$

(a) $x=2$

(b) Removable, assign the value 1 to $f(2)$.

21.

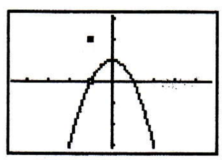


$[-5, 5]$ by $[-4, 8]$

(a) $x=1$

(b) Not removable, it's an infinite discontinuity.

22.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(a) $x=-1$

(b) Removable, assign the value 0 to $f(-1)$.

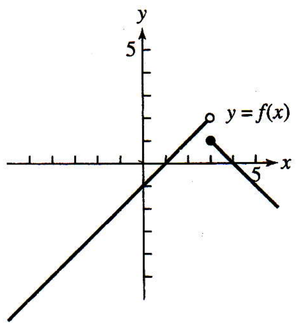
23. (a) All points not in the domain along with $x = 0, 1$

(b) $x = 0$ is a removable discontinuity, assign $f(0) = 0$.
$x = 1$ is not removable, the one-sided limits are different.

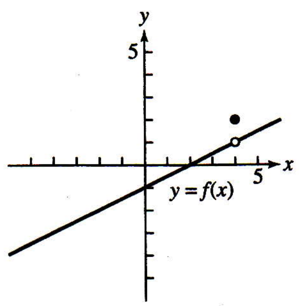
24. (a) All points not in the domain along with $x = 1, 2$

(b) $x = 1$ is not removable, the one-sided limits are different.
$x = 2$ is a removable discontinuity, assign $f(2) = 1$.

41. One possible answer:



43. One possible answer:



47. We require that $\lim_{x \to 3^+} 2ax = \lim_{x \to 3^-} (x^2 - 1)$:

$$2a(3) = 3^2 - 1$$
$$6a = 8$$
$$a = \frac{4}{3}$$

49. Solve at $x = -1$

$$f(x) = 4 - x^2 = 4 - (-1)^2 = 4 + 1 = 5$$
$$f(x) = ax + 1 \text{ at } x = 1$$
$$5 = a(1) + 1$$
$$a = 4$$