Section 2.4 Exercises

1. (a) $\frac{\Delta f}{\Delta x}=\frac{f(3)-f(2)}{3-2}=\frac{28-9}{1}=19$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(1)-f(-1)}{1-(-1)}=\frac{2-0}{2}=1$

3. (a) $\frac{\Delta f}{\Delta x}=\frac{f(0)-f(-2)}{0-(-2)}=\frac{1-e^{-2}}{2}\approx 0.432$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(3)-f(1)}{3-1}=\frac{e^3-e}{2}\approx 8.684$

4. (a) $\frac{\Delta f}{\Delta x}=\frac{f(4)-f(1)}{4-1}=\frac{\ln 4-0}{3}=\frac{\ln 4}{3}\approx 0.462$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(103)-f(100)}{103-100}=\frac{\ln 103-\ln 100}{3}=\frac{1}{3}\ln\frac{103}{100}$
$=\frac{1}{3}\ln 1.03\approx 0.0099$

5. (a) $\frac{\Delta f}{\Delta x}=\frac{f(3\pi/4)-f(\pi/4)}{(3\pi/4)-(\pi/4)}=\frac{-1-1}{\pi/2}=-\frac{4}{\pi}\approx -1.273$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(\pi/2)-f(\pi/6)}{(\pi/2)-(\pi/6)}=\frac{0-\sqrt{3}}{\pi/3}=-\frac{3\sqrt{3}}{\pi}\approx -1.654$

9. (a)
$$\lim_{h\to 0}\frac{y(-2+h)-y(-2)}{h}=\lim_{h\to 0}\frac{(-2+h)^2-(-2)^2}{h}$$
$$=\lim_{h\to 0}\frac{4-4h+h^2-4}{h}$$
$$=\lim_{h\to 0}\frac{-4h+h^2}{h}$$
$$=\lim_{h\to 0}(-4+h)$$
$$=-4$$

(b) The tangent line has slope –4 and passes through

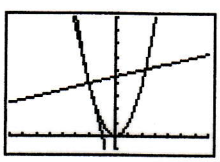
$(-2, y(-2))=(-2, 4)$
$y=-4[x-(-2)]+4$
$y=-4x-4$

(c) The normal line has slope $-\frac{1}{-4}=\frac{1}{4}$ and passes through

$(-2, y(-2))=(-2, 4)$.

$y=\frac{1}{4}[x-(-2)]+4$
$y=\frac{1}{4}x+\frac{9}{2}$

(d)



[–8, 7] by [–1, 9]

11. (a)
$$\lim_{h\to 0}\frac{y(2+h)-y(2)}{h}=\lim_{h\to 0}\frac{\frac{1}{(2+h)-1}-\frac{1}{2-1}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{1}{h+1}-1}{h}$$
$$=\lim_{h\to 0}\frac{1-(h+1)}{h(h+1)}$$
$$=\lim_{h\to 0}\left(-\frac{1}{h+1}\right)$$
$$=-1$$

(b) The tangent line has slope –1 and passes through

$(2, y(2))=(2, 1)$.

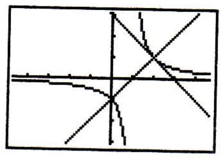
$y=-(x-2)+1$
$y=-x+3$

(c) The normal line has slope $-\frac{1}{-1}=1$ and passes through

$(2, y(2))=(2, 1)$.

$y=1(x-2)+1$
$y=x-1$

(d)



[–4.7, 4.7] by [–3.1, 3.1]

15. First, note that $f(0)=2$.

$$\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^-}\frac{(2-2h-h^2)-2}{h}$$
$$=\lim_{h\to 0^-}\frac{-2h-h^2}{h}$$
$$=\lim_{h\to 0^-}(-2-h)$$
$$=-2$$

$$\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^+}\frac{(2h+2)-2}{h}$$
$$=\lim_{h\to 0^+}2$$
$$=2$$

No, the slope from the left is –2 and the slope from the right is 2. The two-sided limit of the difference quotient does not exist.

17. First, note that $f(2)=\frac{1}{2}$

$$\lim_{h\to 0^-}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0^-}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$$
$$=\lim_{h\to 0^-}\frac{2-(2+h)}{2h(2+h)}$$
$$=\lim_{h\to 0^-}\frac{-h}{2h(2+h)}$$
$$=\lim_{h\to 0^-}-\frac{1}{2(2+h)}$$
$$=-\frac{1}{4}$$

$$\lim_{h\to 0^+}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0^+}\frac{\frac{4-(2+h)}{4}-\frac{1}{2}}{h}$$
$$=\lim_{h\to 0^+}\frac{[4-(2+h)]-2}{4h}$$
$$=\lim_{h\to 0^+}\frac{-h}{4h}$$
$$=-\frac{1}{4}$$

Yes. The slope is $-\frac{1}{4}$.

23. Let $f(t)=100-4.9t^2$.

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$=\lim_{h\to 0}\frac{[100-4.9(2+h)^2]-[100-4.9(2)^2]}{h}$$
$$=\lim_{h\to 0}\frac{100-19.6-19.6h-4.9h^2-100+19.6}{h}$$
$$=\lim_{h\to 0}(-19.6-4.9h)$$
$$=-19.6$$

The object is falling at a speed of 19.6 m/sec.

24. Let $f(t)=3t^2$.

$$\lim_{h\to 0}\frac{f(10+h)-f(10)}{h}=\lim_{h\to 0}\frac{3(10+h)^2-300}{h}$$
$$=\lim_{h\to 0}\frac{300+60h+3h^2-300}{h}$$
$$=\lim_{h\to 0}(60+3h)$$
$$=60$$

The rocket's speed is 60 ft/sec.

29. First, fin the slope of the tangent at $x=a$.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$
$$=\lim_{h\to 0}\frac{[(a+h)^2+4(a+h)-1]-(a^2+4a-1)}{h}$$
$$=\lim_{h\to 0}\frac{a^2+2ah+h^2+4a+4h-1-a^2-4a+1}{h}$$
$$=\lim_{h\to 0}\frac{2ah+h^2+4h}{h}$$
$$=\lim_{h\to 0}(2a+h+4)$$
$$=2a+4$$

The tangent at $x=a$ is horizontal when $2a+4=0$, or $a=-2$. The tangent line is horizontal at $(-2,\ f(-2))=(-2,-5)$.

30. First, find the slope of the tangent at $x=a$.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$
$$=\lim_{h\to 0}\frac{[3-4(a+h)-(a+h)^2]-(3-4a-a^2)}{h}$$
$$=\lim_{h\to 0}\frac{3-4a-4h-a^2-2ah-h^2-3+4a+a^2}{h}$$
$$=\lim_{h\to 0}\frac{-4h-2ah-h^2}{h}$$
$$=\lim_{h\to 0}(-4-2a-h)$$
$$=-4-2a$$

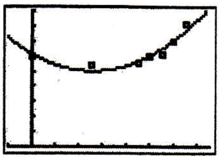
The tangent at $x=a$ is horizontal when $-4-2a=0$, or $a=-2$. The tangent line is horizontal at $(-2, f(-2))=(-2, 7)$.

33. (a) $\frac{(272.1-299.3)\text{ billion}}{(1995-1990)\text{ years}}=\frac{-27.2\text{ billion}}{5\text{ years}}=-\$5.4\frac{\text{billion}}{\text{year}}$

(b) $\frac{(305.5-294.5)\text{ billion}}{(2001-2000)\text{ years}}=\$11.0\frac{\text{billion}}{\text{year}}$

(c) $\frac{(404.9-348.6)\text{ billion}}{(2003-2002)\text{ years}}=\$56.3\frac{\text{billion}}{\text{year}}$

(d) $y\approx 2.177x^2-22.315x+306.443$



[−2, 15] by [0, 450]

(e)

1990 to 1995: $\frac{(2.177(5)^2-22.315(5)+306.443)-(2.177(0)^2-22.315(0)+306.443)}{5-0}$

$=\frac{54.425-111.575+306.443-306.443}{5}$

$=\frac{-57.15}{5}=-\$11.4\text{ billion}$

2000 to 2001: $\frac{(2.177(11)^2-22.315(11)+306.443)-(2.177(10)^2-22.315(10)+306.443)}{11-10}$

$=\frac{263.417-2700.115\text{-}217.7+2231.5}{1}$

$=\$23.4\frac{\text{billion}}{\text{year}}$

2002 to 2003: $\frac{(2.177(13)^2-22.315(13)+306.443)-(2.177(12)^2-22.313(12)+306.443)}{13-12}$

$=367.913-290.095-313.488+267.756$

$=\$32.1\frac{\text{billion}}{\text{year}}$

(f) $\frac{2.177(13.1)^2-22.315(13.1)+306.443-(2.177(13.)^2-22.315(13)+306.443)}{13.1-13}$

$=\frac{373.595-292.327-367.913+290.095}{0.1}=\$34.3\frac{\text{billion}}{\text{year}}$

(g) One possible reason is that the war in Iraq and increased spending to prevent terrorist attacks in the U.S. caused an unusual increase in defense spending.