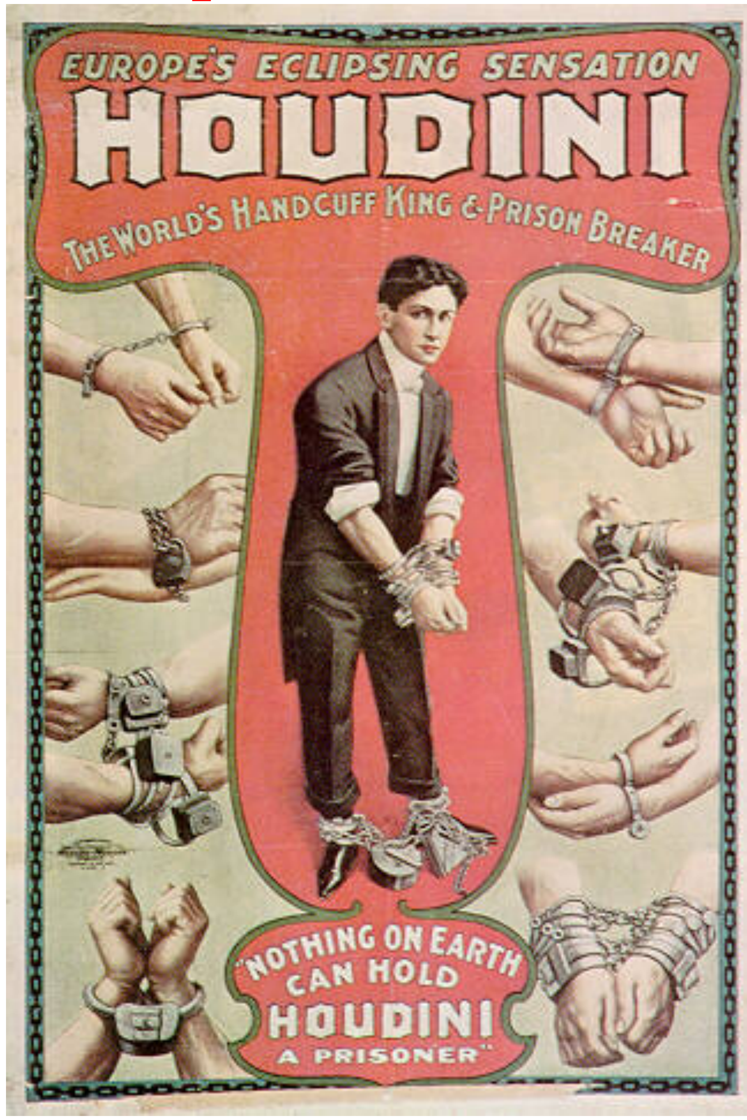
Houdini's Escape



Harry Houdini (1874-1926) was a famous escape artist. In this project we relive a trick of his that challenged his mathematical prowess, as well as his skill and bravery. It will challenge these qualities in you as well.

Houdini had his feet shackled to the top of a concrete block which was placed on the bottom of a giant laboratory flask. The cross-sectional radius of the flask, measured in feet, was given as a function of height z from the ground by the formula

r(z)=(10)/(square-root(z))

with the bottom of the flask at z=1 foot. The flask was then filled with water at a steady rate of 22π cubic feet per minute. Houdini's job was to escape the shackled before he was drowned by the rising water in the flask.

Now Houdini knew it would take him exactly ten minutes to escape the shackles. For dramatic impact, he wanted to time his escape so it was completed precisely at the moment the water level reached the top of his head. Houdini was exactly six feet tall.

In the design of the apparatus, he was allowed to specify only one thing: the height of the concrete block he stood on.

a. Your first task is to find out how high this block should be. Express the volume of water in the flask as a function of the height of the liquid above the ground level. What is the volume when the water level reaches the top of Houdini's head? (Neglect Houdini's volume and the volume of the block) What should the height of the block be?

b. Let h(t) be the height of the water above ground level at time t. In order to check the progress of his escape moment by moment, Houdini derives the equation for the rate of change dh/dt as a function of h(t) itself. Derive this equation. How fast is the water level changing when the flask first starts to fill? How fast is it changing when the water just reaches the top of his head? Express h(t) as a function of time.

c. Houdini would like to be able to perform this trick with any flask. Help him plan his next trick by generalizing the derivation of part (b). Consider a flask with cross sectional radius r(z) (an arbitrary function of z) and a constant inflow rate dV(t)/dt = A. Find dh/dt as a function of h(t).

Extra Credit:
How would you modify your calculations to take into account Houdini's volume, given Houdini's cross-sectional area as a function of height?

HTML Version by Harel Barzilai of Houdin's Escape in *Student Research Projects in Calculus*.