Key

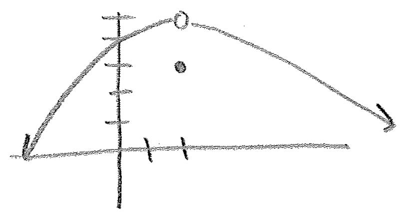
1. **Explain in your own words what is meant by the equation**

$$\lim_{x \to 2} f(x) = 5$$

As x gets close to 2 from the right and left, y gets close to 5.

Is it possible for this statement to be true and yet $f(2) = 3$? Explain

yes. For example:



2. **Explain what it means to say that**

$$\lim_{x \to 1^-} f(x) = 3 \quad \text{and} \quad \lim_{x \to 1^+} f(x) = 7$$

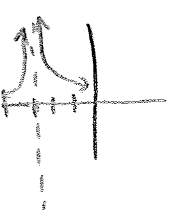
→ As x gets close to 1 from the left, y gets close to 3.

→ As x gets close to 1 from the right, y gets close to 7.

In this situation, is it possible that $\lim_{x \to 1} f(x)$ exists?

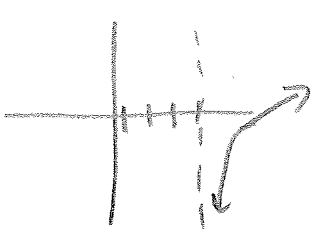
Nope. For $\lim_{x \to 1} f(x)$ to exist $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$.

3. **Explain the meaning of each of the following.**



(a) $\lim_{x \to -3} f(x) = \infty$ As x approaches -3 from both sides, y is going up towards infinity.

(b) $\lim_{x \to 4^+} f(x) = -\infty$ As x approaches 4 from the right, y is going down towards negative infinity

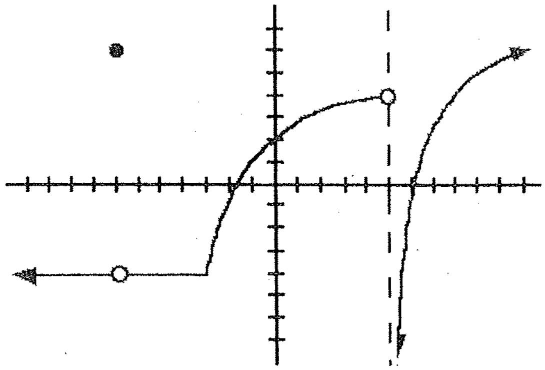


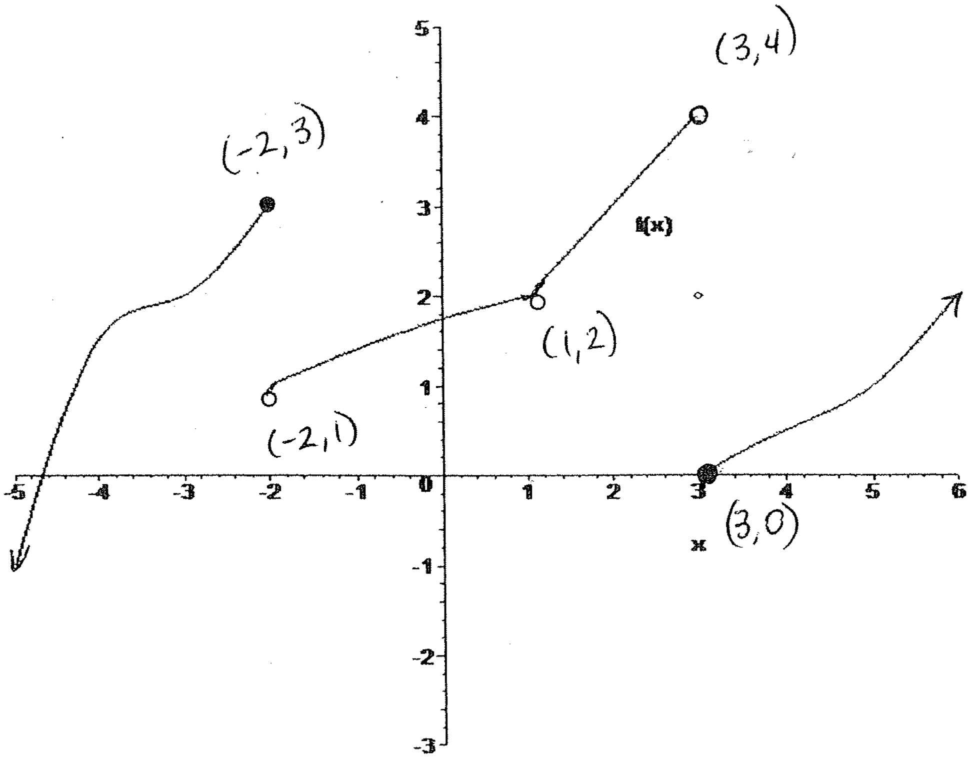
Using the graph below evaluate each limit.

a) Find $\lim_{x \to 5} f(x)$: DNE

b) Find $\lim_{x \to -7} f(x)$: -4

c) Find $\lim_{x \to 0} f(x)$: 2





a) Find $\lim_{x \to -2^-} f(x)$: 3

b) Find $\lim_{x \to -2^+} f(x)$: 1

c) Find $\lim_{x \to -2} f(x)$: DNE

d) Find $\lim_{x \to 1^-} f(x)$: 2

e) Find $\lim_{x \to 1^+} f(x)$: 2

f) Find $\lim_{x \to 3^-} f(x)$: 4

g) Find $\lim_{x \to 3^+} f(x)$: 0

h) Find $\lim_{x \to 3} f(x)$: DNE

Determine the following limits algebraically.
No Calculator.

$$\lim_{x\to 2} \frac{x^2-7x+10}{x^2-4} = \frac{2^2-7(2)+10}{2^2-4} = \frac{0}{0}$$

$$\lim_{x\to 2} \frac{(x-5)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \lim_{x\to 2} \frac{(x-5)}{(x+2)} = \boxed{\frac{-3}{4}}$$

$$\lim_{x\to 5} \frac{x^2+2x-35}{x^2-10x+25} \quad \frac{5^2+2(5)-35}{5^2-10(5)+25} = \frac{0}{0}$$

$$\lim_{x\to 5} \frac{(x+7)\cancel{(x-5)}}{(x-5)\cancel{(x-5)}} = \lim_{x\to 5} \frac{x+7}{x-5} = \frac{5+7}{5-5} = \frac{12}{0}$$

which means $\infty, -\infty$ or DNE

$$\lim_{x\to 5^+} = \frac{5.0001+7}{5.0001-5} = +\infty, \quad \lim_{x\to 5^-} \frac{4.999+7}{4.999-5} = -\infty$$

$\boxed{DNE}$

$$\frac{5-\sqrt{25}}{25-25} = \frac{0}{0}$$

$$\lim_{x\to 25} \frac{5-\sqrt{x}}{25-x} \cdot \frac{5+\sqrt{x}}{5+\sqrt{x}}$$

$$\lim_{x\to 25} \frac{\cancel{(25-x)}}{\cancel{(25-x)}(5+\sqrt{x})} = \lim_{x\to 25} \frac{1}{5+\sqrt{x}} = \frac{1}{5+5} = \boxed{\frac{1}{10}}$$

$$\lim_{x\to 0} \frac{(x+3)^3-27}{x} = \frac{(0+3)^3-27}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{x^3+9x^2+27x+\cancel{27}-\cancel{27}}{x}$$

$$\lim_{x\to 0} \frac{\cancel{x}(x^2+9x+27)}{\cancel{x}} = 0^2+9(0)+27$$

$$= \boxed{27}$$

Determine the following limits at infinity.

$$\lim_{x \to -\infty} \frac{x+7}{3x+5} = \frac{1}{3}$$

$$\lim_{x \to -\infty} \frac{7}{x^3 - 20} = 0$$

$$\lim_{x \to \infty} \frac{100}{x^2 + 5} = 0$$

$$\lim_{x \to -\infty} \frac{7x^2 - x + 11}{4 - x} = \infty$$

$$\lim_{x \to \infty} \frac{x^2 - 3x + 7}{x^3 + 10x - 4} = 0$$

$$\lim_{x \to \infty} \frac{x^2 + 1}{2x - 3}. = \infty$$

$$\lim_{x \to \infty} \frac{7x^2 + x - 100}{2x^2 - 5x} = \frac{7}{2}$$