Integration with u–substitution

U Love U Substitution!!

Name: Key

Integrate the following. You must show work (on another sheet of paper) and used u–substitution.

1. $\int 3x^2(10+2x^3)^5\,dx = 3\int u^5 \cdot \frac{1}{6} \cdot du$

$u = 10+2x^3$
$du = 6x^2 \cdot dx$
$\frac{1}{6}du = x^2 dx$

$\frac{1}{2} \cdot \frac{u^6}{6} + C = \frac{u^6}{12} + C$

$= \boxed{\frac{(10+2x^3)^6}{12} + C}$

2. $\int \frac{x+2}{\sqrt{x^2+4x}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{u}} \cdot du = \frac{1}{2}\int u^{-1/2} du$

$u = x^2 + 4x$
$du = (2x+4)dx$
$du = 2(x+2)dx$

$\frac{1}{2} \frac{u^{1/2}}{1/2} + C = \frac{1}{2} \cdot \frac{2}{1}(x^2+4x)^{1/2} + C$

$= \boxed{(x^2+4x)^{1/2} + C}$

3. $\int \sec^4 t \tan t\,dt$

$u = \sec t$
$du = \sec t \tan t\,dt$

$\int u^3 \cdot du = \frac{u^4}{4} + C = \boxed{\frac{\sec^4 x}{4} + C}$

4. $\int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$
$du = -\sin x \cdot dx$

$= -\int \frac{1}{u} du = -\ln u + C$

$= \boxed{-\ln(\cos x) + C}$

5. $\int 5p(8-p^2)^7\,dp = 5\int u^7 \cdot -\frac{1}{2} du$

$u = 8 - p^2$
$du = -2p \cdot dp$
$-\frac{1}{2}du = p \cdot dp$

$= -\frac{5}{2} \cdot \frac{u^8}{8} + C = \boxed{\frac{-5}{16}(8-p^2)^8 + C}$

6. $\int 5t^4 \sin(8+t^5)\,dt = \int \sin u \cdot du$

$u = 8 + t^5$
$du = 5t^4 \cdot dt$

$= \boxed{-\cos(8+t^5) + C}$

(*) 7. $\int \frac{3q}{1+9q^4}\,dq = \int \frac{3q}{1+(3q^2)^2}$

$u = 3q^2$
$du = 6q \cdot dq$
$\frac{1}{2}du = 3q\,dq$

$= \frac{1}{2}\int \frac{1}{1+u^2} \cdot du = \frac{1}{2}\tan^{-1} u + C$

$= \boxed{\frac{1}{2}\tan^{-1}(3q^2) + C}$

8. $\int 3x^3 e^{6x^4}\,dx = \frac{1}{8}\int e^u \cdot du = \frac{1}{8} \cdot e^u + C$

$u = 6x^4$
$du = 24x^3 \cdot dx$
$\frac{1}{8}du = 3x^3 dx$

$= \boxed{\frac{1}{8} e^{6x^4} + C}$

(*) 9. $\cancel{\int \frac{1+s}{\sqrt{1-s^2}}\,ds}$

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10. $\int \frac{1-x^2}{x^3-3x}\,dx = -\frac{1}{3}\int \frac{1}{u} du$

$u = x^3 - 3x$
$du = (3x^2-3)dx$
$du = -3(-x^2+1)dx$
$-\frac{1}{3}du = (1-x^2)dx$

$= -\frac{1}{3}\ln u + C$

$= \boxed{-\frac{1}{3}\ln(x^3-3x) + C}$

Integration Quiz

Good Luck To: Key

Please SHOW ALL WORK!!! You MUST use u-substitution!!!

1. $\cancel{\int \frac{3x^4+8}{12x^3}\,dx =}$

$u = \ln x$
$du = \frac{1}{x}\cdot dx$

2. $\int \frac{dx}{x\ln x} = \int \frac{1}{u}\cdot dx$
$= \ln u + C = \boxed{\ln(\ln x) + C}$

3. $\cancel{\int \frac{\sin^{-1} x^2}{\sqrt{1-x^4}}\,dx =}$

$u = 1+9x^2$
$du = 18x\cdot dx$
$\frac{1}{18}du = x\,dx$

4. $\int \frac{x}{1+9x^2}\,dx = \frac{1}{18}\int \frac{1}{u}\cdot du = \frac{1}{18}\ln u + C$
$= \boxed{\frac{1}{18}\ln(1+9x^2) + C}$

$u = \tan x$
$du = \sec^2 x\cdot dx = \frac{1}{\cos^2 x}\cdot dx$

5. $\int \frac{4e^{\tan x}}{\cos^2 x}\,dx = 4\int \frac{e^{\tan x}}{\cos^2 x}\,dy$
$= 4\int e^u\cdot du$
$= 4e^u + C = \boxed{4e^{\tan x} + C}$

$u = 8x^2 - 6x + 5$
$du = (16x - 6)\,dx$
$du = 2(8x-3)\,dx$
$\frac{1}{2}du = (8x-3)\,dx$
$-\frac{1}{2}du = (3-8x)\,dx$

6. $\int \frac{3-8x}{\sqrt{8x^2-6x+5}}\,dx =$
$= \frac{-1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{-1}{2}\int u^{-1/2}\,du = -\frac{1}{2}\cdot u^{1/2}\cdot\frac{2}{1} + C$
$= \boxed{-(8x^2-6x+5)^{1/2} + C}$

$u = 10 - x^3$
$du = -3x^2\cdot dx$
$-\frac{1}{3}du = x^2\cdot dx$

7. $\int 5x^2\sqrt{10-x^3}\,dx = 5\int x^2\sqrt{10-x^3}\cdot dx$
$= 5\cdot\frac{-1}{3}\int u^{1/2}\cdot du$
$= \frac{-5}{3}u^{3/2}\cdot\frac{2}{3} + C$
$= \boxed{\frac{-10}{9}(10-x^3)^{3/2} + C}$

$u = x^{1/3}$
$du = \frac{1}{3}x^{-2/3}\cdot dx$
$= 3du = \frac{1}{\sqrt[3]{x^2}}\,dx$

8. $\int \frac{\sin\sqrt[3]{x}}{\sqrt[3]{x^2}}\,dx =$
$3\int \sin u\cdot du$
$= -3\cos u + C$
$= -3\cos x^{1/3} + C$
$= \boxed{-3\cos\sqrt[3]{x} + C}$