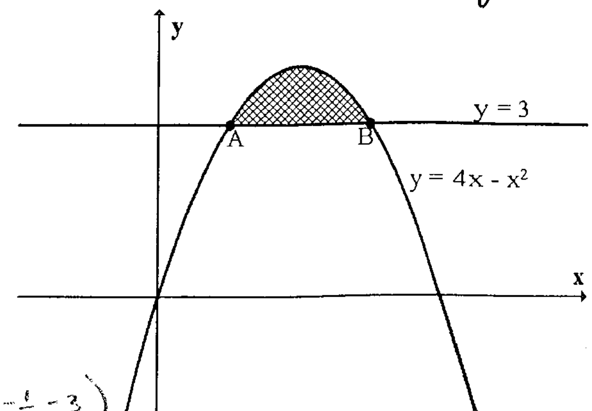
Key

Area Between two Curves

1. The diagram opposite shows the curve $y = 4x - x^2$ and the line $y = 3$.

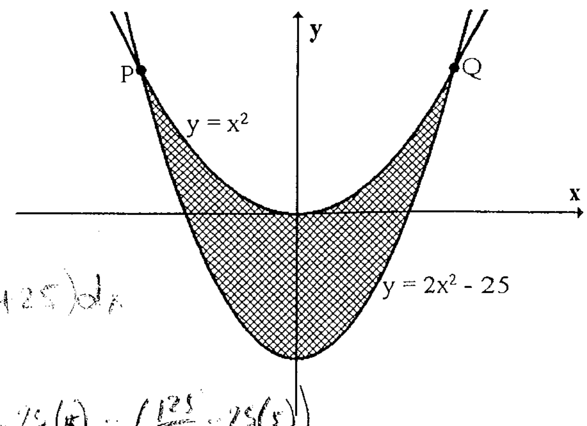


(a) Find the coordinates of A and B.
(b) Calculate the shaded area.

a) $3 = 4x - x^2$
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$x = 3, \quad x = 1$
$\boxed{(3,3)} \quad \boxed{(1,3)}$

b) $\int_1^3 (4x - x^2 - 3)\,dx$
$2x^2 - \frac{x^3}{3} - 3x \Big|_1^3 = (18 - 9 - 9) - (2 - \frac{1}{3} - 3)$
$= \boxed{1\tfrac{1}{3}}$

2. The curves with equations $y = x^2$ and $y = 2x^2 - 25$ intersect at P and Q.

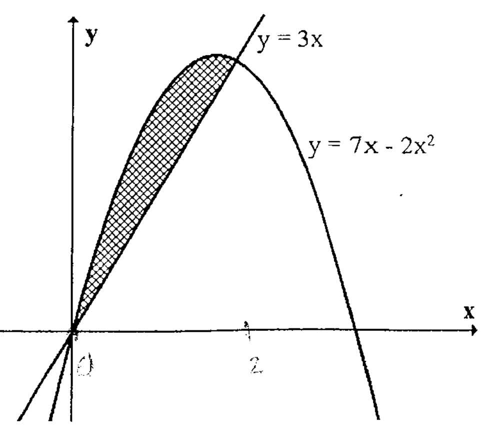


Calculate the area enclosed between the curves.

$x^2 = 2x^2 - 25$
$25 = x^2$
$\pm 5 = x$

$\int_{-5}^{5} (x^2 - 2x^2 + 25)\,dx = \int_{-5}^{5} (-x^2 + 25)\,dx$
$= -\frac{x^3}{3} + 25x \Big|_{-5}^{5} = -\frac{125}{3} + 25(5) - \left(\frac{125}{3} - 25(5)\right)$
$= \boxed{166\tfrac{2}{3}}$

3. The diagram opposite shows the curve $y = 7x - 2x^2$ and the line $y = 3x$.

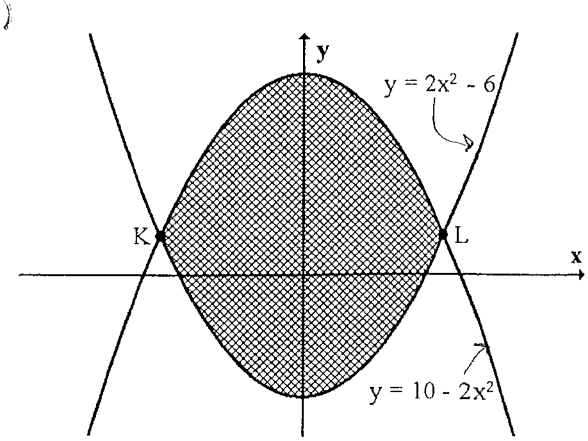


Calculate the shaded area.

$3x = 7x - 2x^2$
$2x^2 - 4x = 0$
$2x(x-2) = 0$
$x = 0 \quad x = 2$

$\int_0^2 (7x - 2x^2 - 3x)\,dx = \int_0^2 (4x - 2x^2)\,dx$
$= 2x^2 - \frac{2x^3}{3} \Big|_0^2 = 8 - \frac{16}{3} - (0)$
$= \boxed{\tfrac{8}{3}}$

4. The curves with equations $y = 2x^2 - 6$ and $y = 10 - 2x^2$ intersect at K and L.



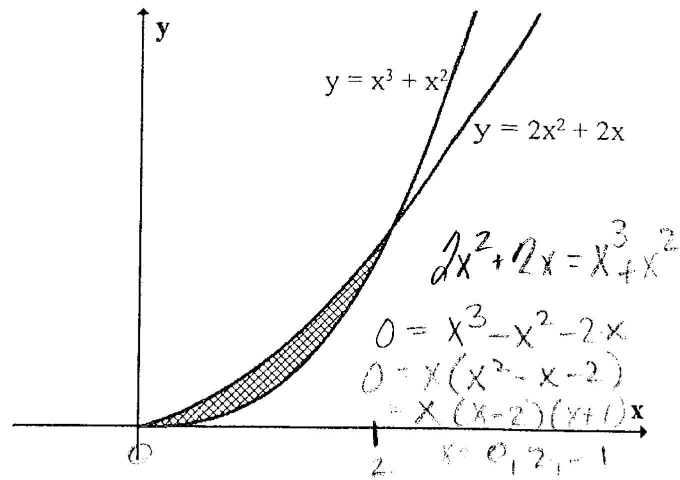
Calculate the area enclosed by these two curves.

$2x^2 - 6 = 10 - 2x^2$
$4x^2 = 16$
$x^2 = 4$
$x = \pm 2$

$\int_{-2}^{2} \left((10 - 2x^2) - (2x^2 - 6)\right)dx$
$\int_{-2}^{2} (16 - 4x^2)\,dx = \boxed{42\tfrac{2}{3}}$

5. The diagram opposite shows part of the curves $y = x^3 + x^2$ and $y = 2x^2 + 2x$.

Calculate the shaded area.



$2x^2 + 2x = x^3 + x^2$

$0 = x^3 - x^2 - 2x$

$0 = x(x^2 - x - 2)$

$= x(x-2)(x+1)$

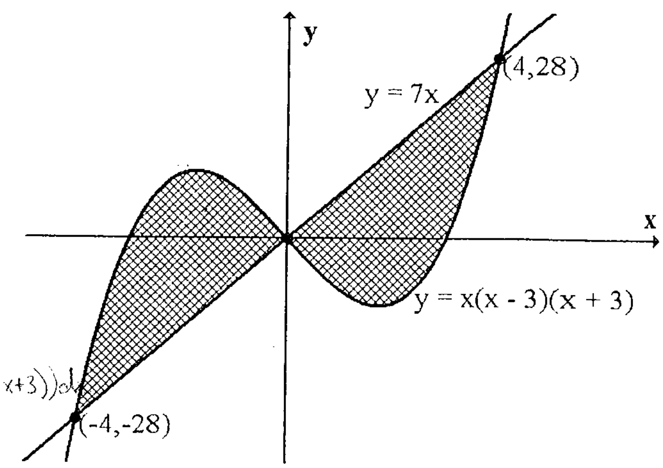
$x = 0, 2, -1$

$= \int_0^2 (2x^2 + 2x - x^3 - x^2)\,dx$

$\int_0^2 (x^2 - x^3 + 2x)\,dx = \boxed{2\tfrac{2}{3}}$

6. The curve $y = x(x - 3)(x + 3)$ and the line $y = 7x$ intersect at the points (0,0), (-4,-28) and (4,28).

Calculate the area enclosed by the curve and the line.



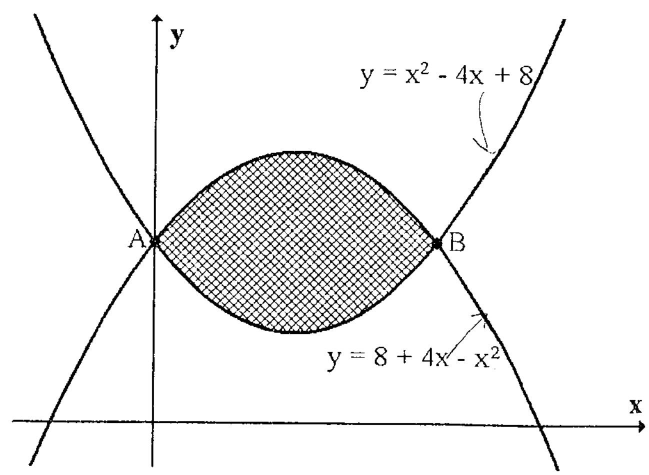
$\int_{-4}^{0} ((x)(x-3)(x+3) - 7x)\,dx + \int_0^4 (7x - x(x-3)(x+3))\,dx$

$= 64 + 64$

$= \boxed{128}$

7. The parabolas $y = x^2 - 4x + 8$ and $y = 8 + 4x - x^2$ intersect at A and B.

(a) Find the coordinates of A and B.
(b) Calculate the shaded area.



a) $x^2 - 4x + 8 = 8 + 4x - x^2$

$2x^2 - 8x = 0$

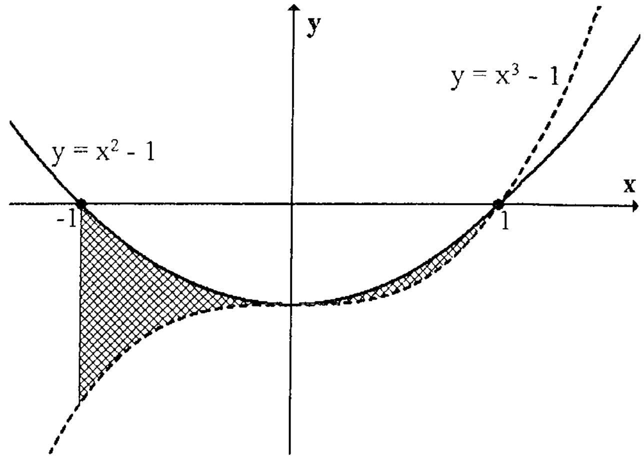
$2x(x - 4) = 0$

$x = 0, x = 4$

b) $\int_0^4 ((8 + 4x - x^2) - (x^2 - 4x + 8))\,dx$

$= \int_0^4 (8x - 2x^2)\,dx = 4x^2 - \frac{2x^3}{3}\Big|_0^4 = 64 - \frac{128}{3} - 0$

$= \frac{192}{3} - \frac{128}{3} = \frac{64}{3} = \boxed{21\frac{1}{3}}$



8. The diagram shows parts of the curves $y = x^3 - 1$ and $y = x^2 - 1$.

Calculate the shaded area.

$x^2 - 1 = x^3 - 1$

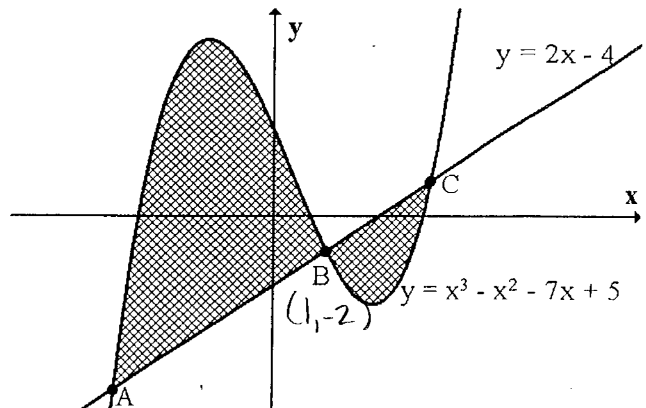
$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0,\ x = 1$

$\int_{-1}^{1}(x^2 - 1 - x^3 + 1)dx$

$= \int_{-1}^{1}(-x^3 + x^2)dx = \boxed{.\overline{6}}$



9. The curve $y = x^3 - x^2 - 7x + 5$ and the line $y = 2x - 4$ are shown opposite.

(a) B has coordinates (1,-2). Find the coordinates of A and C.
(b) Hence calculate the shaded area.

A] $2x - 4 = x^3 - x^2 - 7x + 5$

$0 = x^3 - x^2 - 9x + 9$

$0 = x^2(x-1) - 9(x-1)$

$0 = (x-1)(x^2 - 9)$

$= (x-1)(x+3)(x-3)$ $x = 1, 3, -3$

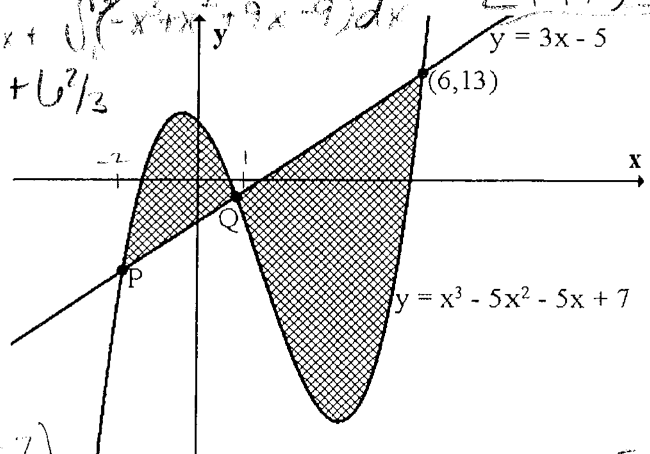
$\boxed{(3,2)}$

$\boxed{(-3,-10)}$

B] $\int_{-3}^{1}(x^3 - x^2 - 7x + 5 - 2x + 4) + \int_{1}^{3}(2x - 4 - x^3 + x^2 + 7x - 5)dx$

$\int_{-3}^{1}(x^3 - x^2 - 9x + 9)dx + \int_{1}^{3}(-x^3 + x^2 + 9x - 9)dx = \boxed{49\frac{1}{3}}$

$42\frac{2}{3} + 6\frac{2}{3}$



10. The diagram shows the line $y = 3x - 5$ and the curve $y = x^3 - 5x^2 - 5x + 7$.

(a) Find the coordinates of P and Q.
(b) Calculate the shaded area.

a] $3x - 5 = x^3 - 5x^2 - 5x + 7$

$0 = x^3 - 5x^2 - 8x + 12$

6	1	-5	-8	12
		6	6	-12
	1	1	-2	0

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2,$

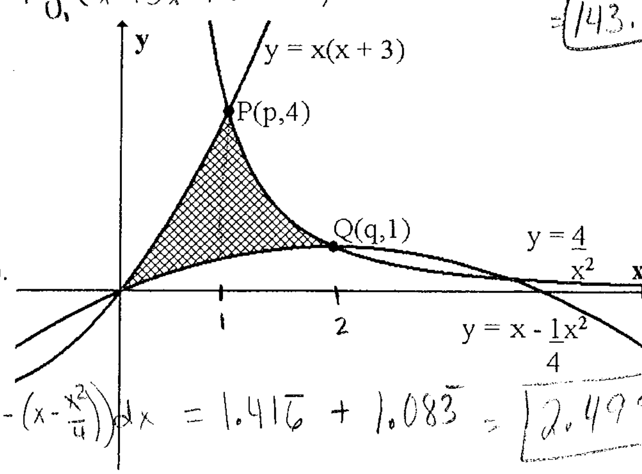
$x = 1,$

b] $\int_{-2}^{1}(x^3 - 5x^2 - 5x + 7 - 3x + 5)dx$

$+ \int_{1}^{6}(3x - 5 - x^3 + 5x^2 + 5x - 7)$

$= \int_{-2}^{1}(x^3 - 5x^2 - 8x + 12)dx + \int_{1}^{6}(-x^3 + 5x^2 + 8x - 12) = 29\frac{1}{4} + 114.58\overline{3}$

$= \boxed{143.8\overline{3}}$



11. The diagram opposite shows an area enclosed by 3 curves:

$$y = x(x+3),\quad y = \frac{4}{x^2}\quad \text{and}\quad y = x - \frac{1}{4}x^2$$

(a) P and Q have coordinates (p,4) and (q,1). Find the values of p and q.

(b) Calculate the shaded area.

A] $x(x+3) = \frac{4}{x^2}$

$x^4 + 3x^3 - 4 = 0$

$x = 1$ $\boxed{P = 1}$

$\frac{4}{x^2} = x - \frac{1}{4}x^2$ $\boxed{Q = 2}$

$4 = x^3 - \frac{1}{4}x^4$

b) $\int_{0}^{1}(x(x+3) - (x - \frac{x^2}{4}))dx + \int_{1}^{2}(\frac{4}{x^2} - (x - \frac{x^2}{4}))dx = 1.41\overline{6} + 1.08\overline{3} = \boxed{2.4999}$