/

Calculus AB CH.2 Review WS 2.1 - 2.5

Name Key Per.____

Match each derivative

1) $f(x)=\tan x$ d
2) $f(x)=\sec x$ c
3) $f(x)=\csc x$ b
4) $f(x)=\sin x$ f
5) $f(x)=\cos x$ a
6) $f(x)=\cot x$ e

a) $f(x)=-\sin x$
b) $f(x)=-\csc x\cot x$
c) $f(x)=\sec x\tan x$
d) $f(x)=\sec^2 x$
e) $f(x)=-\csc^2 x$
f) $f(x)=\cos x$

---

7) $f(x)=3x^2-7x+8$ (2, 6) $y=3(2)^2-7(2)+8$ $y=12-14+8$ $y=6$

a) $f'(x)=6x-7$

b) $f'(2)=6(2)-7=5$

c) Equation of the tangent line at $x=2$

$y-6=5(x-2)$

d) Equation of the normal line at $x=2$

$y-6=\frac{-1}{5}(x-2)$

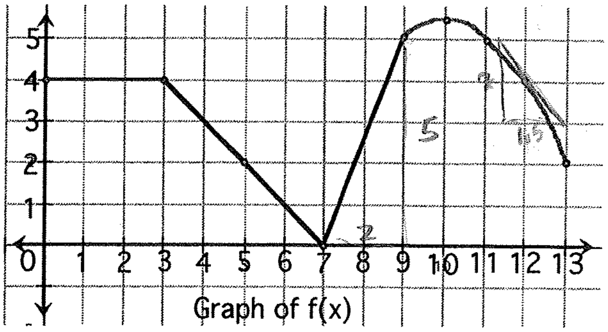
Use picture at right for #8

8 a) $f'(2)=0$ b) $f'(3)=$ DNE c) $f'(5)=-1$

d) $f'(8)=\frac{5}{2}$ e) $f'(9)=$ DNE f) $f'(10)=0$

g) $f'(12)\doteq -1.5$ h) Equation of tangent line at 12 (12, 4)

$y-4=-1.5(x-12)$



Graph of f(x)

---

9) $\lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}=\sec^2 x$ $y=\tan x$

10) $\lim_{h\to 0}\frac{3(x+h)^7-3x^7}{h}=21x^6$ $y=3x^7$

11) $\lim_{h\to 0}\frac{(1+h)^7-1}{h}=7x^6=7(1)^6=\boxed{7}$ $y=x^7$

12) $\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{6}+h\right)-\frac{\sqrt{3}}{2}}{h}=-\sin\frac{\pi}{6}=\frac{-1}{2}$ $x=\frac{\pi}{6}$ $y=\cos x$

Find equation of the tangent line and normal line to the given equation at the given point.

13) $\overset{u}{2x}\overset{v}{y^2}+x=12$; (4,−1)

$2x\cdot 2y\cdot\frac{dy}{dx}+2\cdot y^2+1=0$

$4xy\frac{dy}{dx}=-1-2y^2$

$\frac{dy}{dx}=\frac{-1-2y^2}{4xy}=\frac{-1-2(-1)^2}{4\cdot 4(-1)}$

$=\frac{-3}{-16}=\frac{3}{16}$

tangent line $y+1=\frac{3}{16}(x-4)$

normal line $y+1=\frac{-16}{3}(x-4)$

14) Given: $x^2-6x+y^2+4y-12=0$ and $\frac{dy}{dx}=\frac{-2x+6}{2y+4}$

a) Find the horizontal tangent(s).

$\frac{-2x+6}{2y+4}=0$

$-2x+6=0$

$6=2x$

$3=x$

$3^2-6(3)+y^2+4y-12=0$

$y^2+4y-21=0$

$(y+7)(y-3)=0$

$y=-7, y=3$

b) Find the vertical tangent(s).

$\frac{-2x+6}{2y+4}$

$2y+4=0$

$2y=-4$

$y=-2$

$x^2-6x+(-2)^2+(4)(-2)-12=0$

$x^2-6x-16=0$

$(x-8)(x+2)=0$

$x=8$

$x=-2$

15) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. $x^2 - 9y^2 = 7$

$2x - 18y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-18y}$

$\frac{dy}{dx} = \frac{x}{9y}$

$\frac{d^2y}{dx^2} = \frac{9y \cdot 1 - x \cdot 9\frac{dy}{dx}}{(9y)^2} = \frac{9y - x \cdot \frac{9x}{9y}}{81y^2} = \frac{9y - \frac{x^2}{y}}{81y^2}$

$f(3) = 3$ $f'(3) = 7$ $f(9) = -2$ $f'(9) = 3$

$g(3) = -1$ $g'(3) = 4$ $g(9) = 0$ $g'(9) = 6$

16) Use information above to find $h'(x)$ <u>and</u> $h'(3)$.

a) $h(x) = f(x) \cdot g(x)$

$h'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$h'(3) = 3 \cdot 4 + -1 \cdot 7$

$h'(3) = 12 - 7 = 5$

b) $h(x) = g(f(x))$

$h'(x) = g'(f(x))f'(x)$

$h'(3) = g'(3) \cdot 7$

$= 4 \cdot 7$

$= 28$

c) $h(x) = (f(3x))^3$

$h'(x) = 3(f(3x))^2 \cdot f'(3x) \cdot 3$

$= 3 \cdot (-2)^2 \cdot (3 \times (3))$

$= 108$

<u>Find derivatives for each.</u>

17) $f(x) = \frac{2}{x} = 2x^{-1}$

$f'(x) = -2x^{-2} = \frac{-2}{x^2}$

18) $f(x) = \cos 7x^3$

$f'(x) = -\sin(7x^3) \cdot 21x^2$

$= -21x^2 \sin(7x^3)$

19) $f(x) = x^{7/9}$

$f'(x) = \frac{7}{9}x^{-\frac{2}{9}}$

$= \frac{7}{9x^{2/9}}$

20) $f(x) = \tan(\sec x)$

$f'(x) = \sec^2(\sec x) \cdot \sec x \tan x$

21) $f(x) = \cot 17x$

$f'(x) = -\csc^2(17x) \cdot 17$

$f'(x) = -17\csc^2(17x)$

22) $f(x) = (\sin 5x)^4$

$f'(x) = 4(\sin(5x))^3 \cos(5x) \cdot 5$

$= 20\sin^3(5x) \cdot \cos(5x)$

23) $f(x) = \overset{u}{x^2}\overset{v}{\sqrt{x^2 - 9}}$

$f'(x) = 2x\sqrt{x^2-9} + x^2 \cdot \frac{1}{2}(x^2-9)^{-1/2} \cdot 2x$

$= 2x\sqrt{x^2-9} + \frac{x^3}{\sqrt{x^2-9}}$

24) $f(x) = \overset{u}{3x^3}\overset{v}{(5x^2 + 7)^5}$

$\downarrow$

$f'(x) = 9x^2(5x^2+7)^5 + 3x^3 \cdot 5(5x^2+7)^4 \cdot 10x$

$= 9x^2(5x^2+7)^5 + 150x^4(5x^2+7)^4$

25) $f(x) = \frac{\overset{u}{x^2 + 5}}{\underset{v}{x^3 - 9}}$

$\downarrow$

$f'(x) = \frac{(x^3-9)(2x) - (x^2+5)3x^2}{(x^3-9)^2}$

Calculus CH.3 Review #1 Name: ______________________ Per: ____

Find the extreme values of f on the given interval. Determine at which numbers in the interval they occur.

1) $f(x)=3x^3-9x+4$; $[-2,3]$

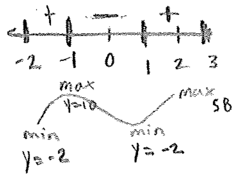
Abs.max.value 58

Abs.min.value -2

Abs.max.occurs at $X=3$

Abs.min.occurs at $X=-2$ or $X=1$

$f'(x)=9x^2-9$
$=9(x^2-1)=0$
$x^2-1=0$
$x=\pm 1$



2) $f(x)=x^{2/5}+3$; $[-32, 1]$

Abs.max.value 7

Abs.min.value 3

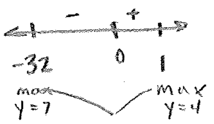
Abs.max.occurs at $X=-32$

Abs.min.occurs at $X=0$

$f'(x)=\frac{2}{5}x^{-\frac{3}{5}}$

$0=\frac{2}{5x^{3/5}}$

No zeros of f'.
undefined x=0



3) Find the relative max. and min. and the intervals on which the given function is increasing and those on which it is decreasing. $f(x)=\overset{u}{x}(\overset{v}{x-10})^3$

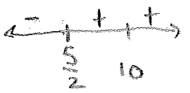
rel.max. NA

rel.min. $x=\frac{5}{2}$

inc. $\left[\frac{5}{2},\infty\right)$

dec. $\left(-\infty,\frac{5}{2}\right]$

$f'(x)=1(x-10)^3+x\cdot 3(x-10)^2$
$=(x-10)^2(x-10+3x)$
$0=(x-10)^2(4x-10)$
$(x-10)^2=0$ $4x-10=0$
$x-10=0$ $4x=10$
$x=10$ $x=\frac{5}{2}$



4) Find any inflection point and the intervals on which the function is concave upward and those on which it is concave downward.

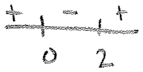
$g(x)=x^4-4x^3+2x+1$

inf. pt. $x=0$, $x=2$

conc.up $(-\infty,0)$ $(2,\infty)$

conc.down $(0,2)$

$g'(x)=4x^3-12x^2+2$
$g''(x)=12x^2-24x$
$0=12x(x-2)$
$x=0$ $x=2$



5) **From $[0,5]$ tell me about the function. (Use graph to the right)**

List the x - coordinates for each:

Inflection points $x=2$

Relative maximum $X=3, 1$

Relative minimum $x=0, 2, 5$

Critical Hard points $X=0, 1, 2, 3, 5$

On which interval(s) is the graph:

increasing/concave up $[0,1]$

increasing/concave down $[2,3]$

decreasing/concave up $[1,2]$

decreasing/concave down $[3,5]$

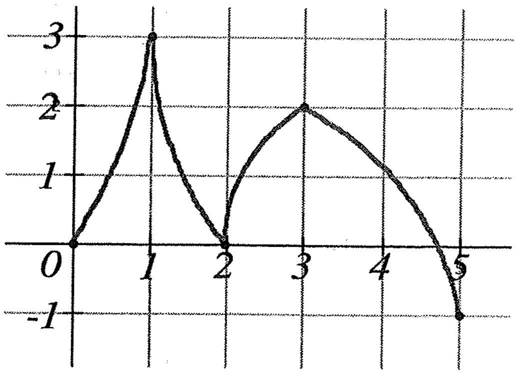
Find each:

Abs. max. value 3

Abs. min. value -1

Abs. max. value occurs at $X=1$

Abs. min. value occurs at $X=5$

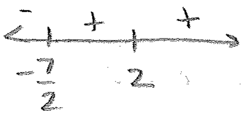


6) If $f(x)=x^3-61$, and $x_1=4$. Use Newton's Method to find the third approximation x_3.

Skip

7) $f''(x)=(x-2)^2(2x+7)$ Find where inflection point(s) occur(s) and concavity.

inf. pt.	conc.up	conc.down
$x=$ $X=\frac{-7}{2}$	$(\frac{-7}{2},\infty)$	$(-\infty,\frac{-7}{2})$

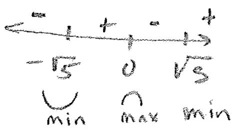


8) $f'(x)=\underset{x}{x^{2}}-\frac{5}{x}$ Find where the rel. extreme values occur and when the graph increases and decreases.

rel.max.	rel.min.	inc.	dec.
$x=0$	$x=\pm\sqrt{5}$	$[-\sqrt{5},0]$ $[\sqrt{5},0]$	$[-\infty,-\sqrt{5}]$ $[0,\sqrt{5}]$

$\frac{x^2-5}{x}\rightarrow x^2-5=0$

$x=\pm\sqrt{5}$ $x=0$



9) The average cost of our product is given by $\overline{C}=10x+\frac{400{,}000}{x}$.

a) How many of our product should we make to minimize the average cost? 200

b) What is the average cost per unit? \$4,000

$\overline{C}'=10-\frac{400{,}000}{x^2}=0$

$10=\frac{400{,}000}{x^2}$

$x^2=\frac{400{,}000}{10}$

$x^2=40{,}000$

$x=200$

$\overline{C}=10(200)+\frac{400{,}000}{200}$

$\overline{C}=\$4000$

Tell me what symbols represent

10) $f'(x)<0$ and $f''(x)>0$ f is decreasing, f is concave up

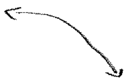
11) $f'(x)>0$ and $f''(x)<0$ f is increasing, f is Concave down

12) $f'(x)<0$ and $f''(x)<0$ f is decreasing, f is concave down

13) $f'(x)>0$ and $f''(x)>0$ f is increasing, f is concave up

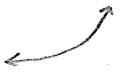
14) **Draw each graph**

dec. /conc. down

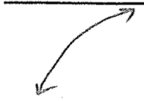


dec. /conc. up

inc. /conc. up



inc. /conc. down



15) Given $f'(-2)=0$ and $f'(5)=0$ and $f''(x)=\frac{9x+11}{(x+3)^3}$. Use the 2nd derivative test to determine if the critical points are relative maximums, relative minimums, or neither.

$f'(c)=0$ + $f''(c)<0$, f local max $x=c$
$f'(c)=0$ + $f''(c)>0$, f local min $x=c$

−2 is relative max

$\frac{9(-2)+11}{(-2+3)^3}=\frac{-7}{1}=-7$

5 is relative min

$\frac{9(5)+11}{(5+3)^3}=\frac{56}{512}$

16) Given $f(x)=\frac{10}{x}$, $10x^{-1}$ find all numbers c in the interval $(1,5)$ where the Mean Value Theorem applies.

$f'(x)=-10x^{-2}$

$=\frac{-10}{x^2}$

$\frac{-10}{x^2}=\frac{f(5)-f(1)}{5-1}$

$\frac{-10}{x^2}=\frac{2-10}{4}$

$\frac{-10}{x^2}=-2$

$\frac{x^2}{-10}=\frac{1}{-2}$

$x^2=5$

$x=\pm\sqrt{5}$

$\boxed{x=+\sqrt{5}}$

CH.2 Related Rates WS

Name: $3(5)^2 - 5y^3 = 35 \rightarrow$ $-5y^3 = -40$ **Per.** $y^3 = 8$ $y = 2$

1) Find $\frac{dx}{dt}$ given $x = 5$ and $\frac{dy}{dt} = 7$ for the equation $3x^2 - 5y^3 = 35$.

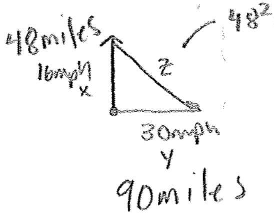
$$6x\frac{dx}{dt} - 15y^2\frac{dy}{dt} = 0$$

$$6 \cdot 5\frac{dx}{dt} - 15 \cdot 4 \cdot 7 = 0$$

$$\frac{dx}{dt} = \frac{420}{30}$$

$$\frac{dx}{dt} = \boxed{14}$$

2) Two trains leave the station at the same time with one train traveling north at 16 mph and the other train traveling east at 30 mph. How fast is the distance between the two trains changing after 3 hours?



$48^2 + 90^2 = z^2$ $z = 102$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\boxed{\frac{dz}{dt} = ?}$$

$$2(48)16 + 2(90)(30) = 2(102)\frac{dz}{dt}$$

$$6936 = 204\frac{dz}{dt}$$

$$\frac{dz}{dt} = \boxed{34 \text{ mph}}$$ ✓

3) The radius of a circle is increasing at the rate of 4 feet per minute.

a) Find the rate at which the area $(A = \pi r^2)$ is increasing when the radius is 12 feet. $96\pi \approx 301.59$ ft²/min

b) Find the rate at which the circumference $(C = 2\pi r)$ is increasing at the same time. $8\pi \approx 25.13$ ft/min

$\frac{dr}{dt} = 4$ ft/min

a)

$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi \cdot 2 \cdot 12 \cdot 4 = 96\pi$$

b)

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$$

$$\frac{dc}{dt} = 2\pi \cdot 4 = 8\pi$$

4) A spherical balloon is inflated at the rate of 11 cubic feet per minute. $\left(V = \frac{4}{3}\pi r^3\right)$

a) How fast is the radius of the balloon changing at the instant the radius is 5 feet? dr/dt? .035 ft/min

b) How fast is the surface area $(A = 4\pi r^2)$ of the balloon changing at the same time? 4.4 ft/min

a)

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3} \cdot \pi \cdot 3 \cdot r^2\frac{dr}{dt}$$

$$11 = 4\pi(5)^2\frac{dr}{dt}$$

$$11 = 4\pi \cdot 25\frac{dr}{dt}$$

$$\frac{11}{100\pi} = \frac{dr}{dt}$$

$$.035 = \frac{dr}{dt}$$

b)

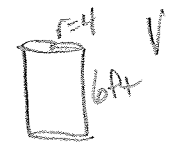
$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 4\pi \cdot 2r\frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi(5)(.035) = 4.4$$

5) The height of a cylinder with a radius of 4 ft. is increasing at a rate of 2 feet per minute.

Find the rate of change of the volume of the cylinder when the height is 6 feet. $(V = \pi r^2 h)$



$V = \pi r^2 h$

$\frac{dh}{dt} = 2$ ft/min

$$\frac{dV}{dt} = ?$$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r\frac{dr}{dt} \cdot h + \pi r^2\frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi(4) \cdot 0 \cdot 6 + \pi 4^2 \cdot 2$$

$$\frac{dV}{dt} = 32\pi \text{ ft}^3/\text{min}$$

6) A conical tank is 20 feet across the top and 15 feet deep. If water is flowing into the tank at the rate of 9 cubic feet per minute, $\frac{dv}{dt} \rightarrow$

$\frac{dh}{dt}$

a) find the rate of change of the depth of the water the instant that it is 2 feet deep. 1.611 ft/min

b) find the rate of change of the surface of the water at the same time. 8.998 ft²/min

a) $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h$

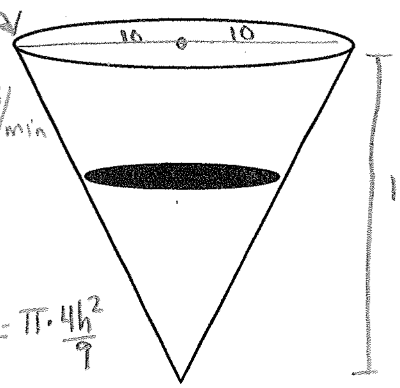
$V = \frac{1}{3}\pi \frac{4h^3}{9}$

$V = \frac{4\pi h^3}{27}$

$\frac{dV}{dt} = \frac{4\pi}{27} \cdot 3h^2 \frac{dh}{dt}$

$9 = \frac{4\pi}{27} \cdot 3(2)^2 \cdot \frac{dh}{dt}$

$1.611 = \frac{dh}{dt}$



$\frac{dv}{dt} = 3ft^3/min$

b) $A = \pi r^2 \rightarrow A = \pi\left(\frac{2}{3}h\right)^2 \rightarrow A = \pi \cdot \frac{4h^2}{9}$

$\frac{dA}{dt} = \frac{4\pi}{9} \cdot 2h \cdot \frac{dh}{dt}$

$= \frac{4\pi}{9} \cdot 2(2)(1.611)$

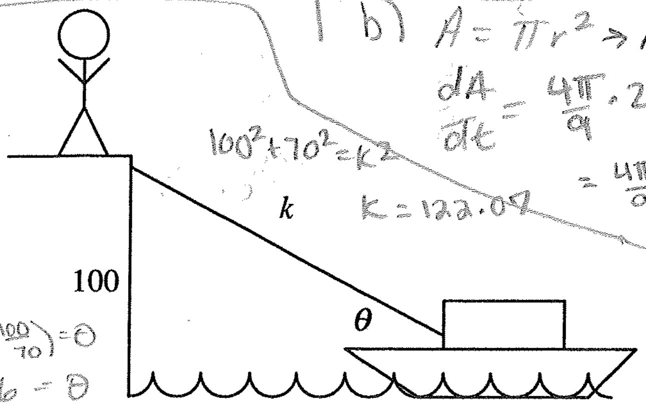
$= 8.998$

$\frac{10}{15} = \frac{r}{h}$

$15r = 10h$

$r = \frac{10}{15}h$

$r = \frac{2}{3}h$



$100^2 + 70^2 = k^2$

$k = 122.07$

$\tan^{-1}\left(\frac{100}{70}\right) = \theta$

$.96 = \theta$

7) A man standing on a 100 ft. cliff watches a boat heading away from the cliff. The boat is travelling at a rate of 88 ft/s. $\frac{dx}{dt}$

a) How fast is the distance *k* between the boat and the man changing when the boat is 70 ft. from the cliff? 50.46 ft/s $\frac{dk}{dt}$

b) How fast is the angle *θ* changing at this time? -.59 rad/sec

a) $100^2 + x^2 = k^2$

$0 + 2x\frac{dx}{dt} = 2k \cdot \frac{dk}{dt}$

$2(70)88 = 2 \cdot 122.07 \cdot \frac{dk}{dt}$

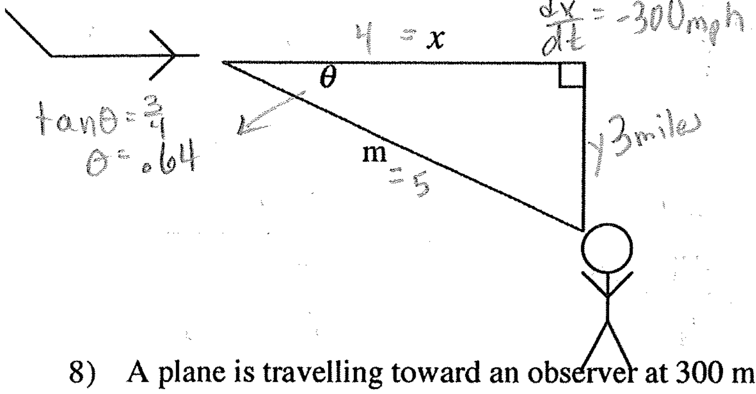
$\frac{dk}{dt} = 50.46$

b) $\tan\theta = \frac{100}{x}$

$\sec^2\theta \frac{d\theta}{dt} = -100x^{-2}\frac{dx}{dt}$

$3.04 \cdot \frac{d\theta}{dt} = -100(70)^{-2} \cdot 88$

$\frac{d\theta}{dt} = -.59$



$\frac{dx}{dt} = -300mph$

$\tan\theta = \frac{3}{4}$

$\theta = .64$

a) $x^2 + 3^2 = m^2$

$2x\frac{dx}{dt} + 0 = 2m\frac{dm}{dt}$

$2 \cdot 4(-300) = 2(5)\frac{dm}{dt}$

$-240 = \frac{dm}{dt}$

8) A plane is travelling toward an observer at 300 mph. The plane is flying 3 miles above the ground.

$\frac{dm}{dt} = ?$ *a*) How fast is the distance *m* between the plane and the man changing when the plane is 5 miles from the man (*m* = 5)? -240mi/h

b) How fast is the angle of depression *θ* changing at this time? 36.18 rad/h

b) $\tan\theta = \frac{3}{x}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = -3x^{-2}\frac{dx}{dt}$

$\sec^2(.64) \cdot \frac{d\theta}{dt} = -3(4)^{-2} \cdot -300$

$1.554\ldots \frac{d\theta}{dt} = 56.25$

$\frac{d\theta}{dt} = 36.18$