Chapter 3 – AP Calc MC Questions (Derivatives)

DEFINITION OF A DERIVATIVE

6. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

B

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

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25. If $f(x)=e^x$, which of the following is equal to $f'(e)$?

E

(A) $\lim_{h\to 0} \dfrac{e^{x+h}}{h}$ (B) $\lim_{h\to 0} \dfrac{e^{x+h}-e^e}{h}$ (C) $\lim_{h\to 0} \dfrac{e^{e+h}-e}{h}$

(D) $\lim_{h\to 0} \dfrac{e^{x+h}-1}{h}$ (E) $\lim_{h\to 0} \dfrac{e^{e+h}-e^e}{h}$

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8. If f is a function such that $\lim_{x\to 2} \dfrac{f(x)-f(2)}{x-2}=0$, which of the following must be true?

C

(A) The limit of $f(x)$ as x approaches 2 does not exist.
(B) f is not defined at $x=2$.
(C) The derivative of f at $x=2$ is 0.
(D) f is continuous at $x=0$.
(E) $f(2)=0$

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29. The $\lim_{h\to 0} \dfrac{\tan 3(x+h)-\tan 3x}{h}$ is

B

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

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10. $\lim_{h\to 0} \dfrac{\sin(x+h)-\sin x}{h}$ is

D

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

---

14. $\lim_{h \to 0} \frac{e^{(2+h)} - e^2}{h} =$

D

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$

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37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

II. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x \to a} \frac{f(x+h) - f(x)}{h}$

C

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

---

79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

C

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

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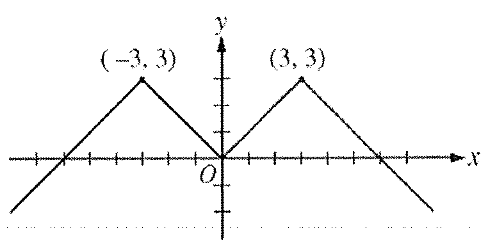

23. The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

B

(A) $\lim_{x \to 0} (f(x) - f(0)) = 0$

(B) $\lim_{x \to 0} \frac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x \to 0} \frac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ does not exist.

IMPLICIT DIFFERENTIATION

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

E

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

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13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

A

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

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40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

E

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

---

9. If $xy^2 + 2xy = 8$, then, at the point $(1, 2)$, y' is

B

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

---

9. If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

E

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

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6. If $y^2 - 2xy = 16$, then $\frac{dy}{dx} =$

C

(A) $\frac{x}{y-x}$ (B) $\frac{y}{x-y}$ (C) $\frac{y}{y-x}$ (D) $\frac{y}{2y-x}$ (E) $\frac{2y}{x-y}$

---

17. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

A

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

---

4. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

A

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17. If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

A

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6. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

A

---

10. If $y=xy+x^2+1$, then when $x=-1$, $\frac{dy}{dx}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

B

---

3. The slope of the line tangent to the curve $y^2+(xy+1)^3=0$ at $(2,-1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

D

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26. What is the slope of the line tangent to the curve $3y^2-2x^2=6-2xy$ at the point $(3,2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

B

---

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

D

---

7. If $y = \ln(x^2 + y^2)$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

D

---

25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

A

DERIVATIVES

18. If $f(x)=2+|x-3|$ for all x, then the value of the derivative $f'(x)$ at $x=3$ is E

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

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1. If $f(x)=x^{\frac{3}{2}}$, then $f'(4)=$ C

(A) -6 (B) -3 (C) 3 (D) 6 (E) 8

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6. If $f(x)=x$, then $f'(5)=$ C

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$

---

23. $\frac{d}{dx}\left(\frac{1}{x^3}-\frac{1}{x}+x^2\right)$ at $x=-1$ is B

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

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4. If $f(x)=-x^3+x+\frac{1}{x}$, then $f'(-1)=$ D

(A) 3 (B) 1 (C) -1 (D) -3 (E) -5

---

10. What is the instantaneous rate of change at $x=2$ of the function f given by $f(x)=\frac{x^2-2}{x-1}$? D

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

---

6. If $f(x)=\frac{x-1}{x+1}$ for all $x\neq -1$, then $f'(1)=$ D

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

---

3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{-6x}{\left(4+x^2\right)^2}$ (B) $\frac{3x}{\left(4+x^2\right)^2}$ (C) $\frac{6x}{\left(4+x^2\right)^2}$ (D) $\frac{-3}{\left(4+x^2\right)^2}$ (E) $\frac{3}{2x}$

A

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4. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (D) $\frac{-5}{(3x+2)^2}$ (E) $\frac{2}{3}$

D

---

4. If u, v, and w are nonzero differentiable functions, then the derivative of $\frac{uv}{w}$ is

(A) $\frac{uv'+u'v}{w'}$ (B) $\frac{u'v'w-uvw'}{w^2}$ (C) $\frac{uvw'-uv'w-u'vw}{w^2}$

(D) $\frac{u'vw+uv'w+uvw'}{w^2}$ (E) $\frac{uv'w+u'vw-uvw'}{w^2}$

E

---

76. If $f(x) = \frac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\frac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\frac{e^{2x}(2x+1)}{x^2}$

(E) $\frac{e^{2x}(2x-1)}{2x^2}$

E

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21. The value of the derivative of $y = \frac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$ at $x = 0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

A

---

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

A

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3. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

D

---

1. If $y = x^2e^x$, then $\dfrac{dy}{dx} =$

(A) $2xe^x$ (B) $x(x+2e^x)$ (C) $xe^x(x+2)$

(D) $2x+e^x$ (E) $2x+e$

C

---

4. If $f(x) = x+\sin x$, then $f'(x) =$

(A) $1+\cos x$ (B) $1-\cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

A

---

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

B

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

---

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

E

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

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8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

E

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

---

4. If $f(x) = \cos^3(4x)$, then $f'(x) =$

B

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

---

8. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

E

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

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10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

D

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

---

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

E

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15. If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

B

---

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

E

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1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

E

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24. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

A

---

5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

C

---

5. If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

(A) $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

(B) $f''(g(x))g'(x) + f'(g(x))g''(x)$

(C) $f''(g(x))[g'(x)]^2$

(D) $f''(g(x))g''(x)$

(E) $f''(g(x))$

A

---

22. $\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

E

---

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\left(f(\ln x)\right) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

A

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31. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

E

---

8. If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

B

---

4. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1 + 2x$ (B) $x + x^2$ (C) $3x^2$ (D) x^3 (E) $x^2 + x^3$

C

---

18. If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\dfrac{1}{1+x^2}$ (B) $\dfrac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$

B

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2. If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

E

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16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

E

---

9. If $y = \cos^2 3x$, then $\dfrac{dy}{dx} =$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

A

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7. $\dfrac{d}{dx}\cos^2(x^3) =$

(A) $6x^2\sin(x^3)\cos(x^3)$

(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

D

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6. If $f(x)=\dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right)=$

(A) 2 (B) $\dfrac{1}{2}$ (C) $1+\dfrac{\pi}{2}$ (D) $\dfrac{\pi}{2}-1$ (E) $1-\dfrac{\pi}{2}$

E

---

8. If $f(x)=e^x$, then $\ln\left(f'(2)\right)=$

(A) 2 (B) 0 (C) $\dfrac{1}{e^2}$ (D) $2e$ (E) e^2

A

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12. If $f(x)=e^{(2/x)}$, then $f'(x)=$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{\left(-2/x^2\right)}$ (D) $-\dfrac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

D

---

25. $\dfrac{d}{dx}\left(2^x\right)=$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\dfrac{2x}{\ln 2}$

C

---

15. If $f(x)=e^{\tan^2 x}$, then $f'(x)=$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x-1}$

(D) $2\tan x\sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

D

---

1. If $f(x)=e^{1/x}$, then $f'(x)=$

(A) $-\dfrac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\dfrac{e^{1/x}}{x}$ (D) $\dfrac{e^{1/x}}{x^2}$ (E) $\dfrac{1}{x}e^{(1/x)-1}$

A

---

36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

A

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n!e^x$

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10. If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$

D

(A) $(\ln 10)10^{(x^2-1)}$ (B) $(2x)10^{(x^2-1)}$ (C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$ (E) $x^2(\ln 10)10^{(x^2-1)}$

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18. $\frac{d}{dx}(\arcsin 2x) =$

D

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

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28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

E

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

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20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

A

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

---

26. If $y = \arctan(e^{2x})$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\frac{2e^{2x}}{1+e^{4x}}$ (C) $\frac{e^{2x}}{1+e^{4x}}$ (D) $\frac{1}{\sqrt{1-e^{4x}}}$ (E) $\frac{1}{1+e^{4x}}$

B

---

9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

A

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3. If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

B

---

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

D

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11. $\frac{d}{dx} \ln\left(\frac{1}{1-x}\right) =$

(A) $\frac{1}{1-x}$ (B) $\frac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

A

---

17. If $f(x) = x\ln\left(x^2\right)$, then $f'(x) =$

(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$ (D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

B

---

31. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x\ln x}$

E

---

28. $\frac{d}{dx}\ln\left|\cos\left(\frac{\pi}{x}\right)\right|$ is

(A) $\frac{-\pi}{x^2\cos\left(\frac{\pi}{x}\right)}$ (B) $-\tan\left(\frac{\pi}{x}\right)$ (C) $\frac{1}{\cos\left(\frac{\pi}{x}\right)}$

(D) $\frac{\pi}{x}\tan\left(\frac{\pi}{x}\right)$ (E) $\frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$

E

---

18. If $y=\cos^2 x-\sin^2 x$, then $y'=$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x+\sin x)$ (E) $2(\cos x-\sin x)$

C

---

24. If $\sin x = e^y$, $0<x<\pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

C

---

8. If $y=\sin x$ and $y^{(n)}$ means "the nth derivative of y with respect to x," then the smallest positive integer n for which $y^{(n)}=y$ is

(A) 2 (B) 4 (C) 5 (D) 6 (E) 8

B

---

14. If $y=x^2+2$ and $u=2x-1$, then $\frac{dy}{du}=$

(A) $\frac{2x^2-2x+4}{(2x-1)^2}$ (B) $6x^2-2x+4$ (C) x^2

(D) x (E) $\frac{1}{x}$

D

---

39. If $y=\tan u$, $u=v-\frac{1}{v}$, and $v=\ln x$, what is the value of $\frac{dy}{dx}$ at $x=e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

D

---

45. If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

D

(A) $f\left(x^6\right)$ (B) $g\left(x^3\right)$ (C) $3x^2 g\left(x^3\right)$

(D) $9x^4 f\left(x^6\right) + 6x\, g\left(x^3\right)$ (E) $f\left(x^6\right) + g\left(x^3\right)$

---

39. Let f and g be differentiable functions such that

$f(1) = 2$, $f'(1) = 3$, $f'(2) = -4$,

$g(1) = 2$, $g'(1) = -3$, $g'(2) = 5$.

D

If $h(x) = f\left(g(x)\right)$, then $h'(1) =$

(A) -9 (B) -4 (C) 0 (D) 12 (E) 15

---

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	-3	4

D

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) -28 (B) -16 (C) 40 (D) 44 (E) 47

TANGENT AND NORMAL LINES

20. An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

A

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

---

8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

B

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

---

3. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

B

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

---

11. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is

A

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x + 1$

(D) $y = 2x - 3$ (E) $y = 7x - 8$

---

32. An equation of the line <u>normal</u> to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

E

(A) $4x + y = -10$ (B) $x - 4y = 23$ (C) $4x - y = 2$ (D) $x + 4y = 25$ (E) $x + 4y = -25$

---

6. Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

B

---

16. The slope of the line <u>normal</u> to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

B

---

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

B

---

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

E

---

11. If $x + 7y = 29$ is an equation of the line <u>normal</u> to the graph of f at the point $(1, 4)$, then $f'(1) =$

(A) 7 (B) $\frac{1}{7}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{29}$ (E) -7

A

---

7. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

B

---

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

D

---

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$
(B) $y = 7x + 7$
(C) $y = 7x + 11$
(D) $y = -5x - 1$
(E) $y = -5x - 5$

C

---

89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$
(B) $y - 3 = -5(x - 2)$
(C) $y - 6 = -5(x - 2)$
(D) $y - 6 = -7(x - 2)$
(E) $y - 6 = -10(x - 2)$

D

---

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

A

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

---

11. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

B

(A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

---

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

C

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

---

12. At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

(A) $\left(\frac{1}{2},-\frac{1}{2}\right)$ (B) $\left(\frac{1}{2},\frac{1}{8}\right)$ (C) $\left(1,-\frac{1}{4}\right)$ (D) $\left(1,\frac{1}{2}\right)$ (E) $(2,2)$

B

---

80. Let f be the function given by $f(x)=2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

A

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

---

77. Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

C

(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258

---

6. The line normal to the curve $y=\sqrt{16-x}$ at the point $(0,4)$ has slope

A

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) −8

---

18. In the xy-plane, the line $x+y=k$, where k is a constant, is tangent to the graph of $y=x^2+3x+1$. What is the value of k?

A

(A) −3 (B) −2 (C) −1 (D) 0 (E) 1

---

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y=\frac{1}{x}+k$ (where k takes all real values)?

D

(A) $y=-x$ (B) $y=-x^2$ (C) $y=-\frac{1}{3}x^3$ (D) $y=\frac{1}{3}x^3$ (E) $y=\ln x$

---

11. If the line $3x-4y=0$ is tangent in the first quadrant to the curve $y=x^3+k$, then k is

B

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

---

4. For what non-negative value of b is the line given by $y=-\frac{1}{3}x+b$ normal to the curve $y=x^3$?

(A) 0 (B) 1 (C) $\frac{4}{3}$ (D) $\frac{10}{3}$ (E) $\frac{10\sqrt{3}}{3}$

C

---

19. A curve has slope $2x+3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$

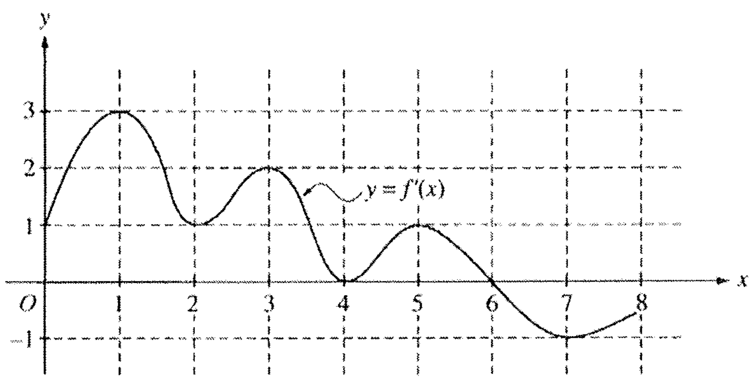
(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

D



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

C

7. The point $(3,5)$ is on the graph of $y=f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y=2$

(B) $y=5$

(C) $y-5=2(x-3)$

(D) $y+5=2(x-3)$

(E) $y+5=2(x+3)$

PARTICLE MOTION

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

C

---

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

B

---

25. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

(A) No values (B) 0 only (C) $\frac{1}{2}$ only (D) 1 only (E) 0 and $\frac{1}{2}$

C

---

16. A particle moves along the x-axis so that at any time $t \ge 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

C

---

12. The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \ge 0$. When $t = \pi$, the acceleration of the particle is

(A) 9 (B) $\frac{1}{9}$ (C) 0 (D) $-\frac{1}{9}$ (E) -9

E

---

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

C

---

79. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

D

---

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

E

---

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

C

---

76. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t + 2)$. What is the acceleration of the particle at time $t = 6$?

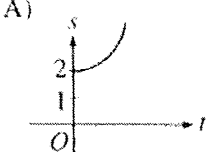
(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

C

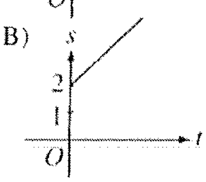
---

90. A particle starts from rest at the point $(2, 0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

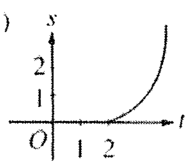
(A)



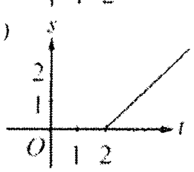
(B)



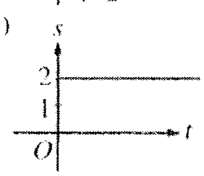
(C)



(D)



E)



A

DERIVATIVES OF INVERSES

40. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2)=5$ and $f'(5)=-\frac{1}{2}$, then $g'(-2)=$

E

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

---

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

B

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

---

28. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

A

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

---

90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

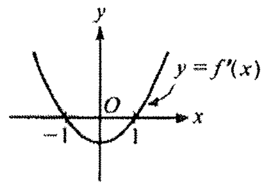
E

(C) $g(8) = 3$ and $g'(8) = -9$

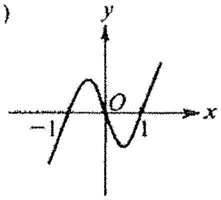
(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

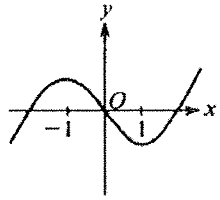
(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

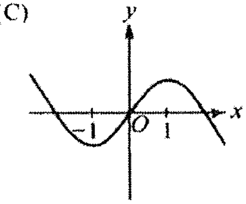
GRAPHS OF DERIVATIVES

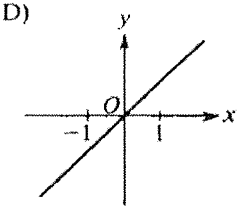


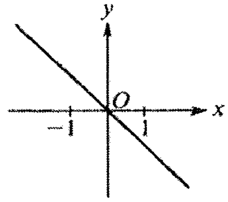
33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


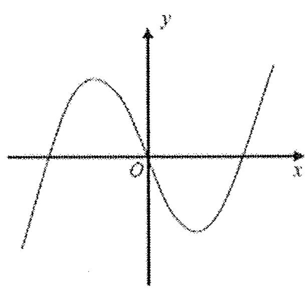
(B)


(C)


(D)


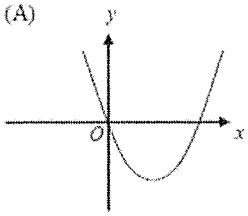
(E)


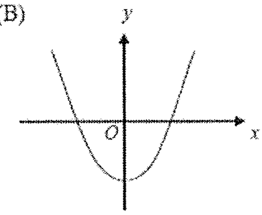
B

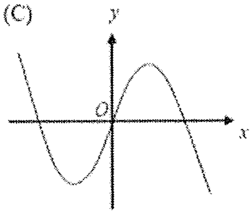


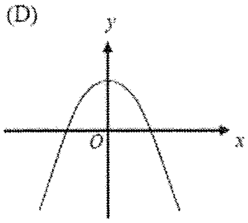
Graph of f

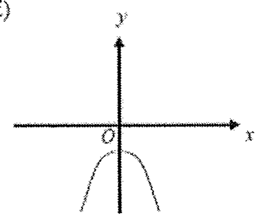
11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

(A)


(B)


(C)


(D)


(E)


B

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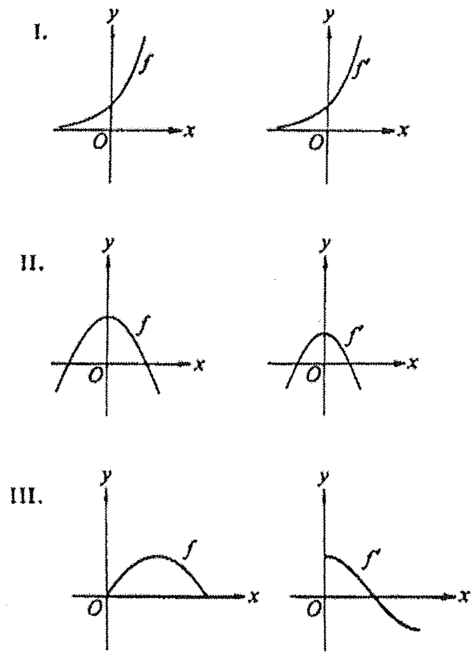
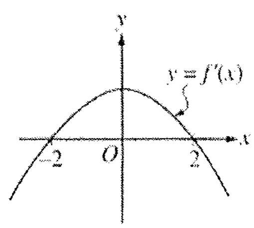
9. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?



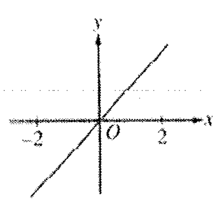
(A) I only (B) II only (C) III only (D) I and III (E) II and III

D

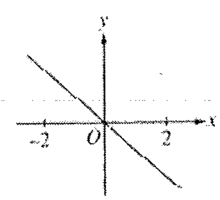


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

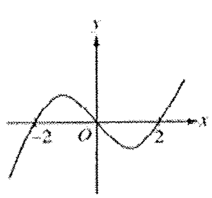
(A)



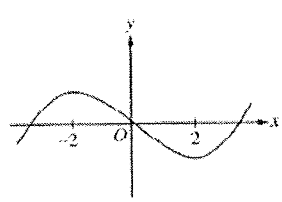
(B)



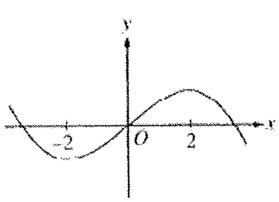
(C)



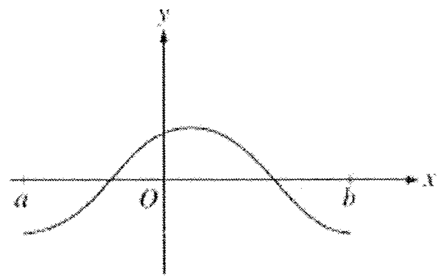
(D)



(E)

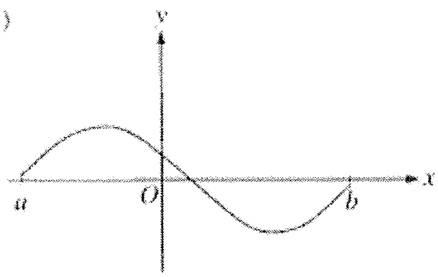


E

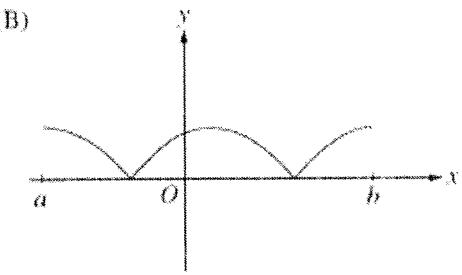


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

(A)

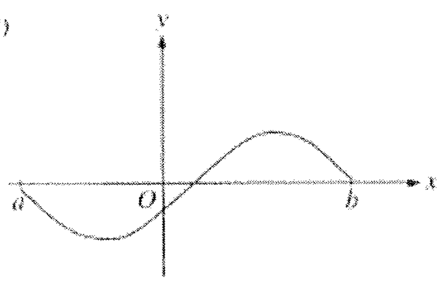


(B)

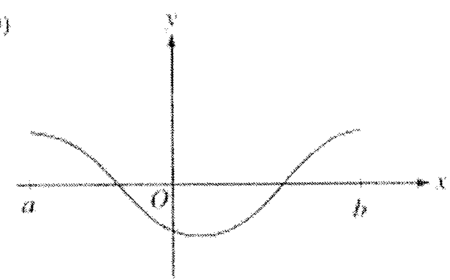


A

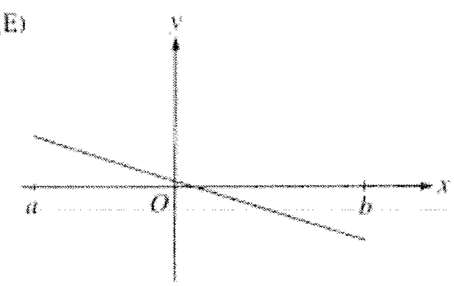
(C)

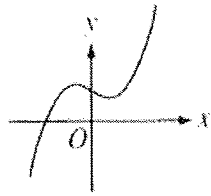


(D)



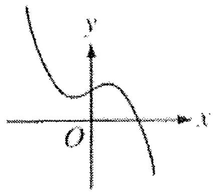
(E)



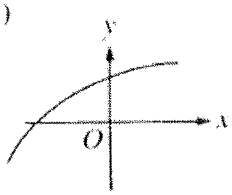


6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

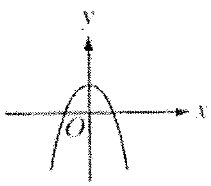
(A)



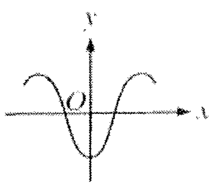
(B)



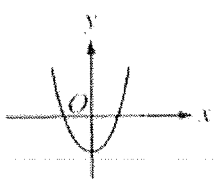
(C)



(D)



(E)



E

CONTINUOUS / DIFFERENTIABLE

41. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

A

(A) None (B) II only (C) III only

(D) I and III only (E) I, II, and III

---

27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

E

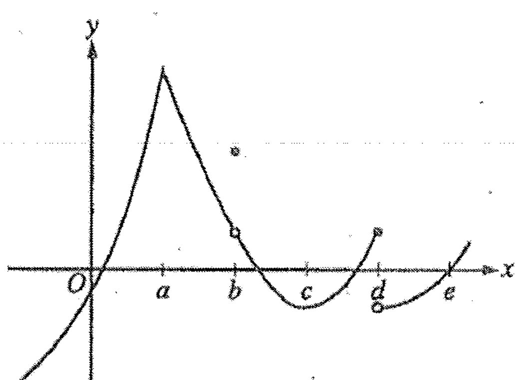
(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

---

16. Which of the following functions shows that the statement "If a function is continuous at $x = 0$, then it is differentiable at $x = 0$" is false?

C

(A) $f(x) = x^{-\frac{4}{3}}$ (B) $f(x) = x^{-\frac{1}{3}}$ (C) $f(x) = x^{\frac{1}{3}}$ (D) $f(x) = x^{\frac{4}{3}}$ (E) $f(x) = x^3$



Graph of f

A

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

D

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \ne 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

A

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

B

(A) −4 (B) −2 (C) 0 (D) 2 (E) 4

---

80. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$.

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$.

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

B

---

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

E

---

82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

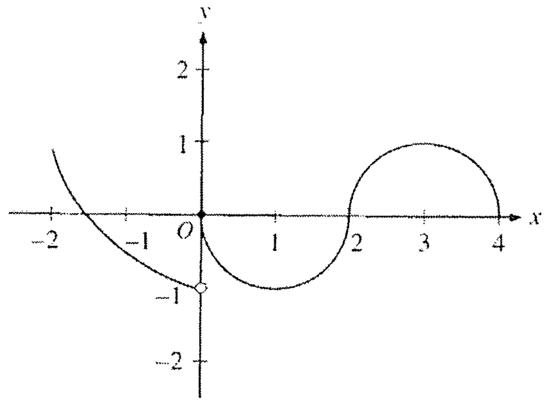
(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \ge f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

C



B

13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

MISCELLANEOUS

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$

(B) $-f'(x_0)$

(C) $\dfrac{1}{f'(x_0)}$

(D) $\dfrac{-1}{f'(x_0)}$

(E) None of the above

A

---

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.

(B) intersect no more than once.

(C) do not intersect.

(D) could intersect more than once.

(E) have a common tangent at each point of intersection.

B

---

86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\dfrac{1}{4}$ (B) 1 (C) 4 (D) $\dfrac{1}{\sqrt{2}}$ (E) $\dfrac{1}{2\sqrt{2}}$

A

---

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

E

---

91. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2)=-5$, $f(5)=5$, and $f(9)=-5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2<c<5$, $f(c)=3$.

E

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

---

39. If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

C

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

---

78. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

(A) The change in temperature during the first day
(B) The change in temperature during the 24th hour
(C) The average rate at which the temperature changed during the 24th hour
(D) The rate at which the temperature is changing during the first day
(E) The rate at which the temperature is changing at the end of the 24th hour

E

---

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

D

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

---

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

86. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

B

I. The minimum value of f on $[3, 7]$ is 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III