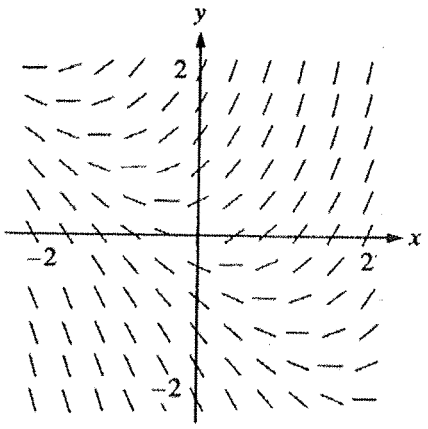
Chapter 6 – AP Calc MC Questions (Integration & Applications)

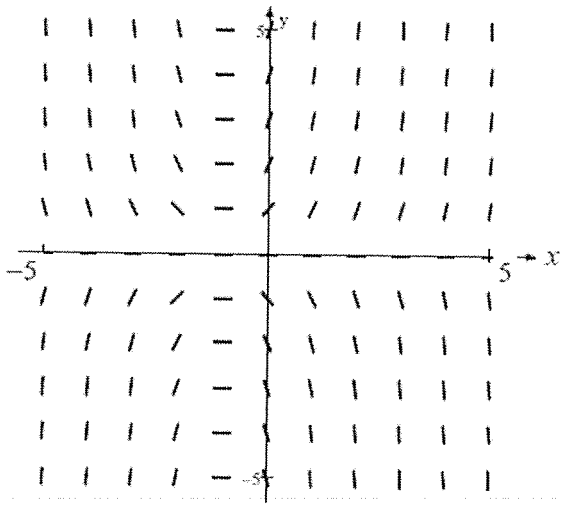
SLOPE FIELDS



C

24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$



C

27. Shown above is a slope field for which of the following differential equations?

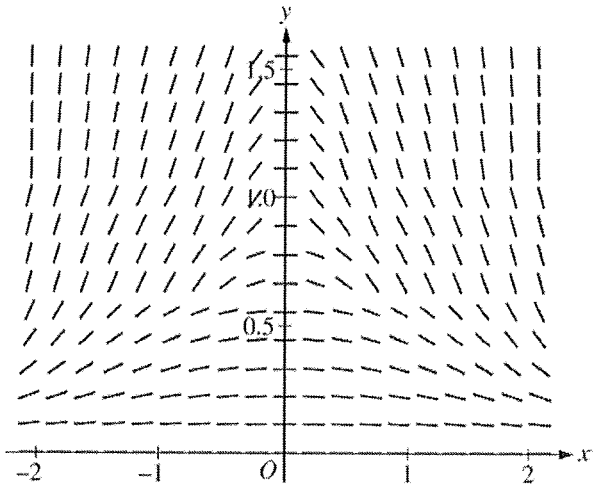
(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$



E

15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

SEPARATION OF VARIABLES

37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

A

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

---

44. At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0, 8)$, then its equation is

A

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

---

39. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

C

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$

(D) $\tan x + 5$ (E) $\tan x + 5e^x$

---

13. If $\frac{dy}{dx} = x^2y$, then y could be

C

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

---

33. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

B

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

---

83. If $\frac{dy}{dx} = (1 + \ln x)y$ and if $y = 1$ when $x = 1$, then $y =$

E

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1 + \ln x$

(C) $\ln x$

(D) $e^{2x+x\ln x-2}$

(E) $e^{x\ln x}$

---

23. Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y}$ with the initial condition $y(3)=-2$?

(A) $y=2e^{-9+x^3/3}$

(B) $y=-2e^{-9+x^3/3}$

E

(C) $y=\sqrt{\frac{2x^3}{3}}$

(D) $y=\sqrt{\frac{2x^3}{3}-14}$

(E) $y=-\sqrt{\frac{2x^3}{3}-14}$

EXPONENTIAL GROWTH & DECAY

16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t=0$, the number of bacteria present was 7,500. Find the number present at $t=5$.

C

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

---

43. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

A

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

---

42. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

B

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

---

38. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

C

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

---

21. If $\frac{dy}{dt}=ky$ and k is a nonzero constant, then y could be

B

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt}+3$ (D) $kty+5$ (E) $\frac{1}{2}ky^2+\frac{1}{2}$

---

84. Population y grows according to the equation $\frac{dy}{dt}=ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

A

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

---

84. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

A

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

---

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N-p)$

(C) $\frac{dp}{dt} = kp(p-N)$

(D) $\frac{dp}{dt} = kt(N-t)$

(E) $\frac{dp}{dt} = kt(t-N)$

B

BASIC INTEGRATION PROBLEMS

43. $\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

D

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

B

---

32. $\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$ (E) $5\ln\left(1+x^2\right)+C$

D

---

2. $\int_0^3 (x+1)^{1/2}\,dx =$

(A) $\frac{21}{2}$ (B) 7 (C) $\frac{16}{3}$ (D) $\frac{14}{3}$ (E) $-\frac{1}{4}$

D

---

1. $\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

D

---

4. If $\frac{dy}{dx}=\cos(2x)$, then $y =$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

C

1. $\int \cos(3x)\,dx =$

(A) $-3\sin(3x) + C$

(B) $-\frac{1}{3}\sin(3x) + C$

(C) $\frac{1}{3}\sin(3x) + C$

(D) $\sin(3x) + C$

(E) $3\sin(3x) + C$

C

---

7. Which of the following is equal to $\ln 4$?

(A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t\,dt$ (D) $\int_1^4 \ln x\,dx$ (E) $\int_1^4 \frac{1}{t}\,dt$

E

---

30. $\int \tan(2x)\,dx =$

(A) $-2\ln\left|\cos(2x)\right| + C$ (B) $-\frac{1}{2}\ln\left|\cos(2x)\right| + C$ (C) $\frac{1}{2}\ln\left|\cos(2x)\right| + C$

(D) $2\ln\left|\cos(2x)\right| + C$ (E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

B

---

32. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

D

---

22. $\int_1^2 \frac{x^2 - 1}{x + 1}\,dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

A

---

36. $\int_{-1}^{1} \frac{3}{x^2}\,dx$ is

(A) -6 (B) -3 (C) 0 (D) 6 (E) nonexistent

E

---

7. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}\,dx$?

(A) $\arcsin\frac{x}{5}+C$ (B) $\arcsin x+C$ (C) $\frac{1}{5}\arcsin\frac{x}{5}+C$

(D) $\sqrt{25-x^2}+C$ (E) $2\sqrt{25-x^2}+C$

A

---

17. $\int_0^1 (3x-2)^2\,dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

D

---

17. $\int (x^2+1)^2\,dx =$

(A) $\frac{(x^2+1)^3}{3}+C$

(B) $\frac{(x^2+1)^3}{6x}+C$

(C) $\left(\frac{x^3}{3}+x\right)^2+C$

(D) $\frac{2x(x^2+1)^3}{3}+C$

(E) $\frac{x^5}{5}+\frac{2x^3}{3}+x+C$

E

---

13. $\int\left(x^3+1\right)^2\,dx =$

(A) $\frac{1}{7}x^7+x+C$

(B) $\frac{1}{7}x^7+\frac{1}{2}x^4+x+C$

(C) $6x^2\left(x^3+1\right)+C$

(D) $\frac{1}{3}\left(x^3+1\right)^3+C$

(E) $\frac{\left(x^3+1\right)^3}{9x^2}+C$

B

---

2. $\int_0^1 x\left(x^2+2\right)^2\,dx =$

D

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$

---

1. $\int_0^1 \sqrt{x}\,(x+1)\,dx =$

C

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

---

3. $\int_1^2 \frac{1}{x^2}\,dx =$

C

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

---

7. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

E

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

---

6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

C

(A) $e^{-t}+C$ (B) $e^{-\frac{t}{2}}+C$ (C) $e^{\frac{t}{2}}+C$ (D) $2e^{\frac{t}{2}}+C$ (E) e^t+C

---

2. $\int_0^1 e^{-4x}\,dx =$

D

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4}-1$ (D) $\frac{1}{4}-\frac{e^{-4}}{4}$ (E) $4-4e^{-4}$

---

4. $\int \left(\sin(2x)+\cos(2x)\right)dx =$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

B

---

7. $\int_2^{+\infty} \frac{dx}{x^2}$ is

(A) $\frac{1}{2}$ (B) $\ln 2$ (C) 1 (D) 2 (E) nonexistent

A

---

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

D

---

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

A

---

25. $\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

B

U-SUB INTEGRATION PROBLEMS

38. $\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\frac{1}{3}\ln e^{x^3}+C$ (B) $-\frac{e^{x^3}}{3}+C$ (C) $-\frac{1}{3e^{x^3}}+C$

(D) $\frac{1}{3}\ln e^{x^3}+C$ (E) $\frac{x^3}{3e^{x^3}}+C$

C

---

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

B

---

27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx =$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

E

---

3. $\int_1^2 \frac{x+1}{x^2+2x}\,dx =$

(A) $\ln 8-\ln 3$ (B) $\frac{\ln 8-\ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2+2}{2}$

B

---

20. $\int x\sqrt{4-x^2}\,dx =$

(A) $\frac{\left(4-x^2\right)^{3/2}}{3}+C$ (B) $-\left(4-x^2\right)^{3/2}+C$ (C) $\frac{x^2\left(4-x^2\right)^{3/2}}{3}+C$

(D) $-\frac{x^2\left(4-x^2\right)^{3/2}}{3}+C$ (E) $-\frac{\left(4-x^2\right)^{3/2}}{3}+C$

E

---

19. $\int_2^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

B

---

7. $\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

(A) $\frac{1}{9}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (B) $\frac{1}{4}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (C) $\frac{1}{12}\left(3x^2+5\right)^{\frac{1}{2}}+C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{\frac{1}{2}}+C$ (E) $\frac{3}{2}\left(3x^2+5\right)^{\frac{1}{2}}+C$

D

---

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

D

---

14. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

A

---

7. $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\frac{1}{4}(e-1)$ (B) $\frac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

A

---

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

C

---

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2e^{\sqrt{x}} + C$

(B) $\frac{1}{2}e^{\sqrt{x}} + C$

(C) $e^{\sqrt{x}} + C$

(D) $2\sqrt{x}\,e^{\sqrt{x}} + C$

(E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

A

---

15. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4\left(x^2-4\right)^2} + C$

(B) $\frac{1}{2\left(x^2-4\right)} + C$

(C) $\frac{1}{2}\ln\left|x^2-4\right| + C$

(D) $2\ln\left|x^2-4\right| + C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

C

---

8. $\int x^2\cos\left(x^3\right)dx =$

(A) $-\frac{1}{3}\sin\left(x^3\right) + C$

(B) $\frac{1}{3}\sin\left(x^3\right) + C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right) + C$

(D) $\frac{x^3}{3}\sin\left(x^3\right) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

B

---

8. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

B

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

---

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

A

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

---

28. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}} dx$, the resulting integral is

C

(A) $\int_0^{1/2} \sin^2 y \, dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y} dy$ (C) $2\int_0^{\pi/4} \sin^2 y \, dy$

(D) $\int_0^{\pi/4} \sin^2 y \, dy$ (E) $2\int_0^{\pi/6} \sin^2 y \, dy$

---

40. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x} dx =$

A

(A) $\int_1^2 \frac{1-u^2}{u} du$ (B) $\int_2^4 \frac{1-u^2}{u} du$ (C) $\int_1^2 \frac{1-u^2}{2u} du$

(D) $\int_1^2 \frac{1-u^2}{4u} du$ (E) $\int_2^4 \frac{1-u^2}{2u} du$

---

11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\, dx$ is equivalent to

C

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\, du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\, du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\, du$ (D) $\int_0^2 \sqrt{u}\, du$ (E) $\int_1^5 \sqrt{u}\, du$

OTHER INTEGRATION PROBLEMS

26. $\int_0^1 \sqrt{x^2-2x+1}\,dx$ is

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) none of the above

C

---

24. If $\int_{-2}^{2}\left(x^7+k\right)dx=16$, then $k=$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

D

---

10. If $\int_0^k\left(2kx-x^2\right)dx=18$, then $k=$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

C

---

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx=\int_0^1(1-x)^n\,dx$ for

(A) no n (B) n even, only (C) n odd, only

(D) nonzero n, only (E) all n

E

---

38. For $x>0$, $\int\left(\frac{1}{x}\int_1^x\frac{du}{u}\right)dx=$

(A) $\frac{1}{x^3}+C$ (B) $\frac{8}{x^4}-\frac{2}{x^2}+C$ (C) $\ln(\ln x)+C$

(D) $\frac{\ln\left(x^2\right)}{2}+C$ (E) $\frac{(\ln x)^2}{2}+C$

E

---

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

C

---

28. $\int_1^{500}\left(13^x-11^x\right)dx+\int_2^{500}\left(11^x-13^x\right)dx=$

B

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

---

90. Which of the following are antiderivatives of $f(x)=\sin x\cos x$?

I. $F(x)=\dfrac{\sin^2 x}{2}$

II. $F(x)=\dfrac{\cos^2 x}{2}$

D

III. $F(x)=\dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

---

22. An antiderivative for $\dfrac{1}{x^2-2x+2}$ is

(A) $-(x^2-2x+2)^{-2}$

(B) $\ln(x^2-2x+2)$

E

(C) $\ln\left|\dfrac{x-2}{x+1}\right|$

(D) $\operatorname{arcsec}(x-1)$

(E) $\arctan(x-1)$

AVERAGE VALUE

44. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0,2]$ is

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

A

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27. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

A

---

20. The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\frac{\sin 5 - \sin 3}{8}$

(B) $\frac{\sin 5 - \sin 3}{2}$

(C) $\frac{\sin 3 - \sin 5}{2}$

(D) $\frac{\sin 3 + \sin 5}{2}$

(E) $\frac{\sin 3 + \sin 5}{8}$

E

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83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

A

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91. What is the average value of $y = \frac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) −0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

C

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88. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1 + 5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

C

(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833

PARTICLE MOTION APPLICATIONS

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

A

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

---

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

B

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

---

14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

D

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

---

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

C

(A) −45 (B) −30 (C) −15 (D) −10 (E) −5

---

15. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at $t = 0$ its position is 4, then at any time t its position $x(t)$ is

C

(A) $t^2 - 4t$ (B) $t^2 - 4t - 4$ (C) $t^2 - 4t + 4$ (D) $2t^2 - 4t$ (E) $2t^2 - 4t + 4$

---

3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

A

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

---

91. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

E

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

---

11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

C

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20. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t > 0$ is $x(t) =$

(A) $-\frac{e^{-2t}}{2} + 3$

(B) $\frac{e^{-2t}}{4} + 4$

(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$

(E) $\frac{e^{-2t}}{4} + 3t + 4$

E

---

91. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 5te^{-t} - 1$. At $t = 0$, the particle is at position $x = 1$. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

(A) 0.366 (B) 0.542 (C) 1.542 (D) 1.821 (E) 2.821

D

MISCELLANEOUS

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

C

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41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$

D

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37. If $f(x) = \begin{cases} x & \text{for } x \leq 1 \\ \frac{1}{x} & \text{for } x > 1, \end{cases}$ then $\int_{0}^{e} f(x)dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e + \frac{1}{2}$

B

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82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \leq t \leq 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

A

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79. A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668

A
