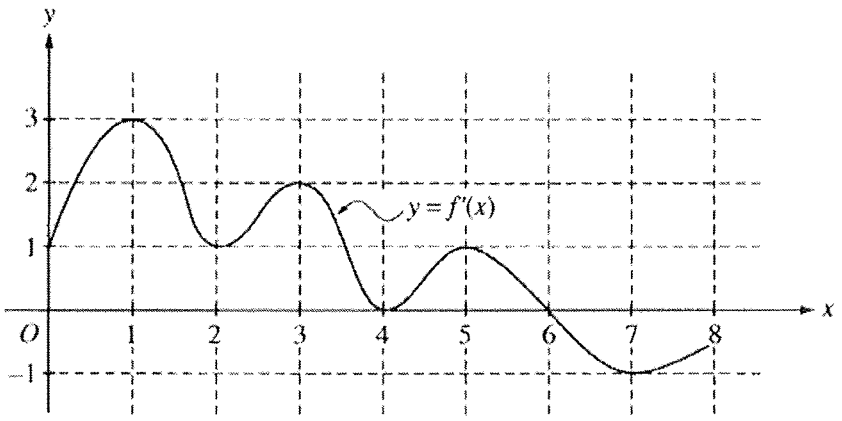
## Chapter 7 – AP Calc MC Questions (Integration Applications)

## READING VELOCITY GRAPHS & PARTICLE MOTION




E

The function $f$ is defined on the closed interval $[0,8]$. The graph of its derivative $f'$ is shown above.

8. How many points of inflection does the graph of $f$ have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

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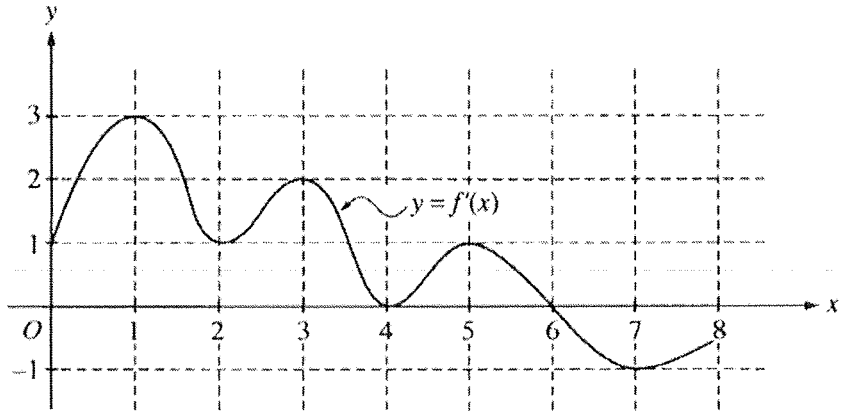


The function $f$ is defined on the closed interval $[0,8]$. The graph of its derivative $f'$ is shown above.

9. At what value of $x$ does the absolute minimum of $f$ occur?

A

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

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26. A particle moves along a line so that at time $t$, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

D

(A) –5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

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13. A particle moves along the $x$-axis so that its acceleration at any time $t$ is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time $t$ during the interval $0 \le t \le 4$ is the particle farthest to the right?

B

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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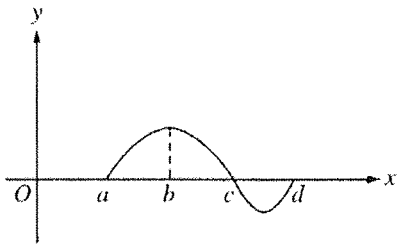


22. The graph of $f$ is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of $x$ does $g(x)$ have a maximum?

C

(A) $a$
(B) $b$
(C) $c$
(D) $d$
(E) It cannot be determined from the information given.

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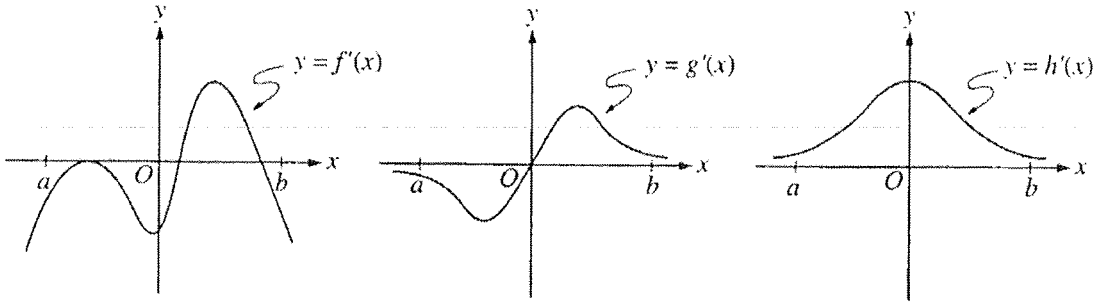
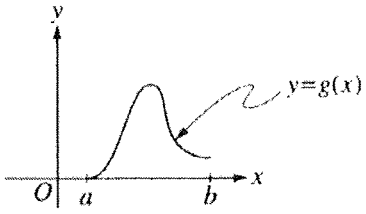



79. The graphs of the derivatives of the functions $f$, $g$, and $h$ are shown above. Which of the functions $f$, $g$, or $h$ have a relative maximum on the open interval $a < x < b$?

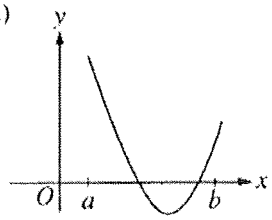
A

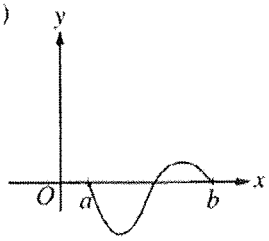
(A) $f$ only
(B) $g$ only
(C) $h$ only
(D) $f$ and $g$ only
(E) $f$, $g$, and $h$

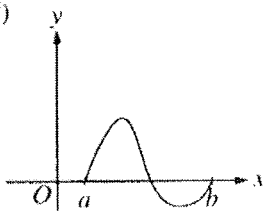


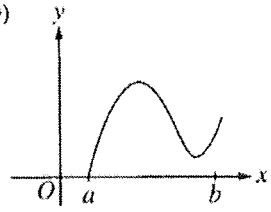

88. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of $g$ on $[a,b]$. Which of the following could be the graph of $f$ on $[a,b]$?

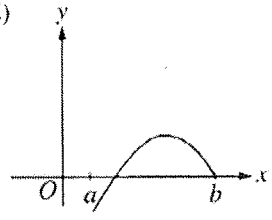
C

(A)



(B)



(C)



(D)



(E)
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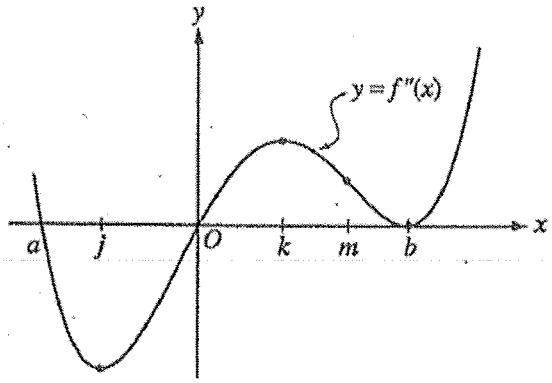


A

21. The second derivative of the function $f$ is given by $f''(x) = x(x-a)(x-b)^2$. The graph of $f''$ is shown above. For what values of $x$ does the graph of $f$ have a point of inflection?

(A) 0 and $a$ only (B) 0 and $m$ only (C) $b$ and $j$ only (D) 0, $a$, and $b$ (E) $b$, $j$, and $k$

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76. A particle moves along the $x$-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

C

(A) −2.016 (B) −0.677 (C) 1.633 (D) 1.814 (E) 2.978

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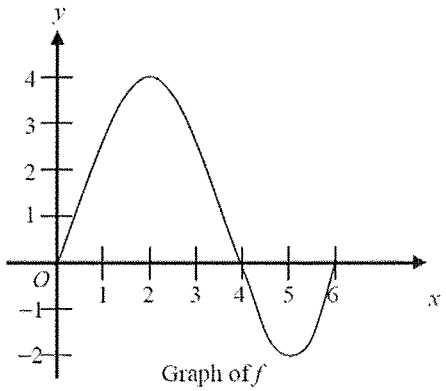

Graph of $f$

17. The graph of the function $f$ shown above has horizontal tangents at $x = 2$ and $x = 5$. Let $g$ be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of $x$ does the graph of $g$ have a point of inflection?

C

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

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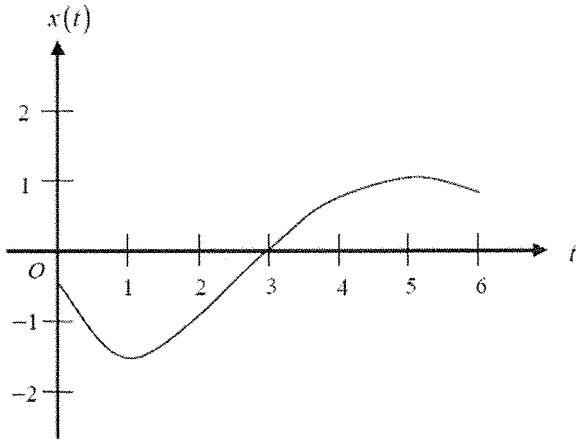


21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time $t$ is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of $t$ is the velocity of the particle increasing?

A

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

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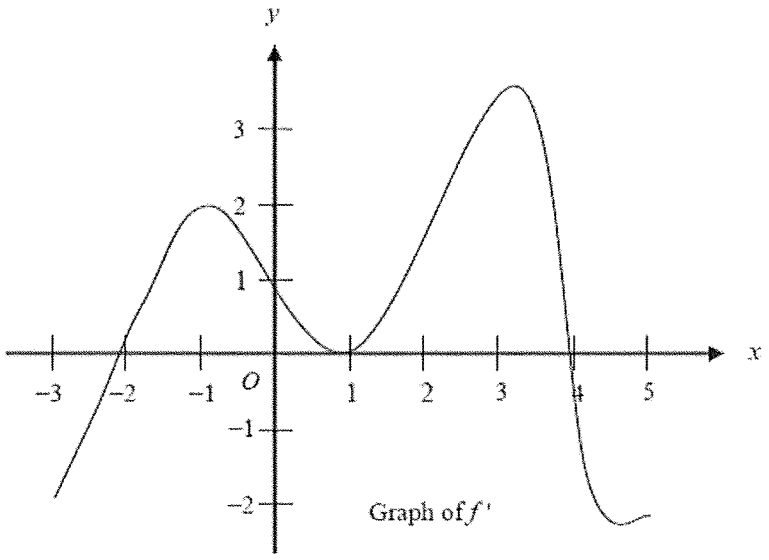


84. The graph of the derivative of a function $f$ is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of $x$ does $f$ have a relative maximum?

C

(A) $-2$ only

(B) 1 only

(C) 4 only

(D) $-1$ and 3 only

(E) $-2$, 1, and 4

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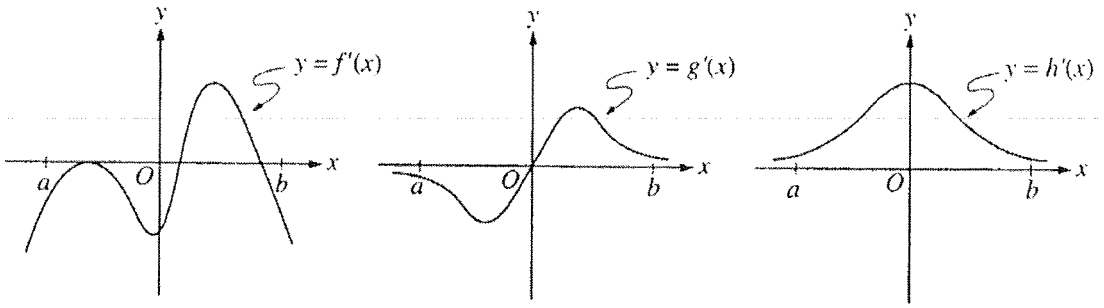


79. The graphs of the derivatives of the functions $f$, $g$, and $h$ are shown above. Which of the functions $f$, $g$, or $h$ have a relative maximum on the open interval $a < x < b$?

A

(A) $f$ only
(B) $g$ only
(C) $h$ only
(D) $f$ and $g$ only
(E) $f$, $g$, and $h$

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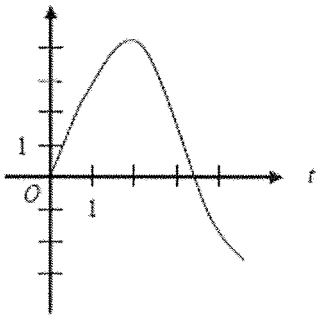
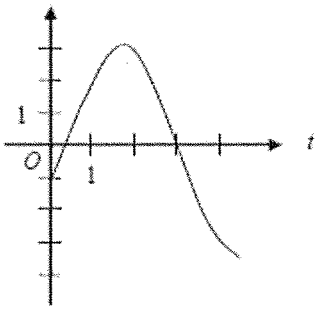
| $t$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $v(t)$ | –1 | 2 | 3 | 0 | –4 |

86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the $x$-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

(A) $x(t)$

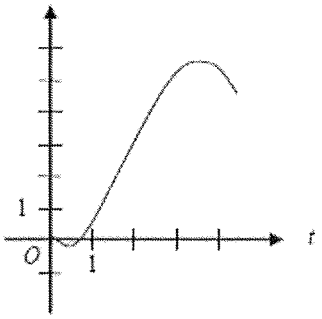



(B) $x(t)$

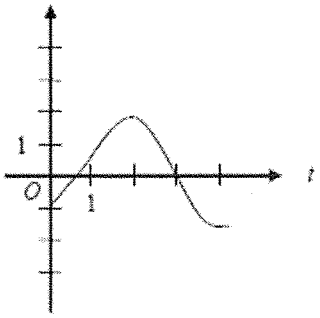



C

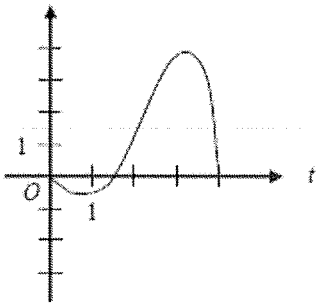
(C) $x(t)$




(D) $x(t)$




(E) $x(t)$

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87. An object traveling in a straight line has position $x(t)$ at time $t$. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1+t^2}$, what is the position of the object at time $t = 3$?

D

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

---

82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

B

(A) $-0.914$ (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

---

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 (-7 + f(t))\,dt$

E

(E) $-7 + \int_2^4 f(t)\,dt$

---

# AREA BETWEEN TWO CURVES

23. The area of the region bounded by the curve $y = e^{2x}$, the $x$-axis, the $y$-axis, and the line $x = 2$ is equal to

C

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

---

17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

C

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

---

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

D

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) $8\pi$

---

34. The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

A

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$

---

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

C

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

---

6. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the $x$-axis, and the lines $x = 3$ and $x = 4$ is

D

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

---

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the $y$-axis?

C

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

---

1. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

A

---

1. The area of the region between the graph of $y = 4x^3 + 2$ and the $x$-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9

D

---

17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$ ?

(A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

D

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

C

---

1. The area of the region in the first quadrant enclosed by the graph of $y = x(1-x)$ and the $x$-axis is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) 1

A

---

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

D

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83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667

(B) 11.833

(C) 14.583

(D) 21.333

(E) 32

B

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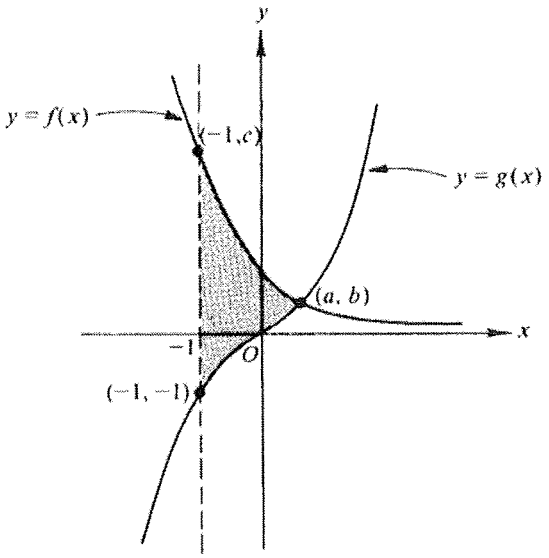


5. The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point $(a, b)$. The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

D

(A) $\int_0^a \big(f(x) - g(x)\big)\,dx + \int_{-1}^0 \big(f(x) + g(x)\big)\,dx$

(B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_{-1}^c \big(f(x) - g(x)\big)\,dx$

(D) $\int_{-1}^a \big(f(x) - g(x)\big)\,dx$

(E) $\int_{-1}^a \big(|f(x)| - |g(x)|\big)\,dx$

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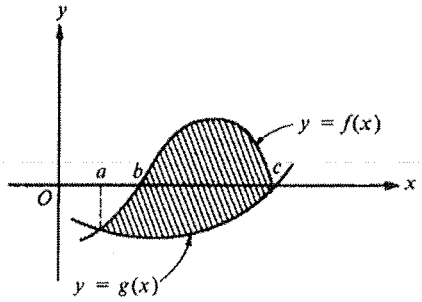


34. The area of the shaded region in the figure above is represented by which of the following integrals?

D

(A) $\int_a^c \big(|f(x)| - |g(x)|\big)\,dx$

(B) $\int_b^c f(x)\,dx - \int_a^c g(x)\,dx$

(C) $\int_a^c \big(g(x) - f(x)\big)\,dx$

(D) $\int_a^c \big(f(x) - g(x)\big)\,dx$

(E) $\int_a^b \big(g(x) - f(x)\big)\,dx + \int_b^c \big(f(x) - g(x)\big)\,dx$

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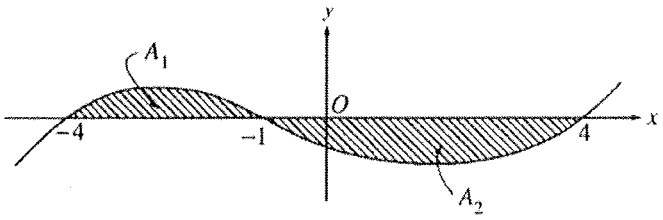


D

19. The graph of $y = f(x)$ is shown in the figure above. If $A_1$ and $A_2$ are positive numbers that represent the areas of the shaded regions, then in terms of $A_1$ and $A_2$,

$\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) $A_1$ (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

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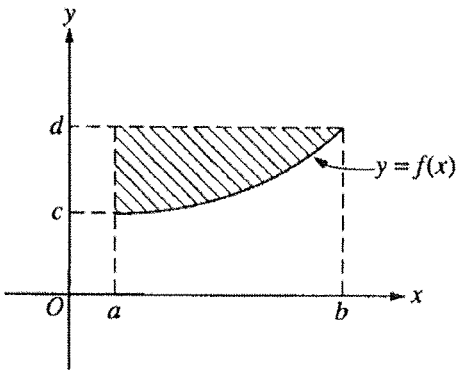


2. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$ (B) $\int_a^b (d - f(x))dx$ (C) $f'(b) - f'(a)$

B

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

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13. The region bounded by the $x$-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

C

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

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25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the $x$-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of $m$.
(B) increases as $m$ increases.
(C) decreases as $m$ increases.
(D) decreases as $m$ increases when $m < \frac{1}{2}$; increases as $m$ increases when $m > \frac{1}{2}$.
(E) increases as $m$ increases when $m < \frac{1}{2}$; decreases as $m$ increases when $m > \frac{1}{2}$.

A

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92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

D

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# CROSS SECTIONS

39. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the $x$-axis are squares, then its volume is

A

(A) $\dfrac{\left(1-e^{-6}\right)}{2}$ (B) $\dfrac{1}{2}e^{-6}$ (C) $e^{-6}$ (D) $e^{-3}$ (E) $1-e^{-3}$

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25. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the $x$-axis. Each plane section of the solid perpendicular to the $x$-axis is a square. The volume of the solid is

D

(A) $\dfrac{4\pi}{3}$ (B) $\dfrac{16\pi}{5}$ (C) $\dfrac{4}{3}$ (D) $\dfrac{16}{5}$ (E) $\dfrac{64}{5}$

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84. The base of a solid $S$ is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the $x$-axis. If the cross sections of $S$ perpendicular to the $x$-axis are squares, then the volume of $S$ is

C

(A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$ (C) 1 (D) 2 (E) $\dfrac{1}{3}(e^3-1)$

---

87. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the $y$-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$

(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

B

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right)dx$

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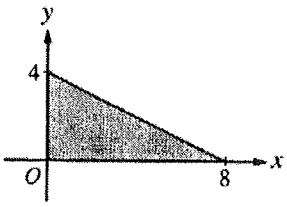


C

86. The base of a solid is a region in the first quadrant bounded by the $x$-axis, the $y$-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the $x$-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

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86. The base of a solid is the region in the first quadrant bounded by the $y$-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the $x$-axis is a square. What is the volume of the solid?

B

(A) 2.561 (B) 6.612 (C) 8.046 (D) 8.755 (E) 20.773

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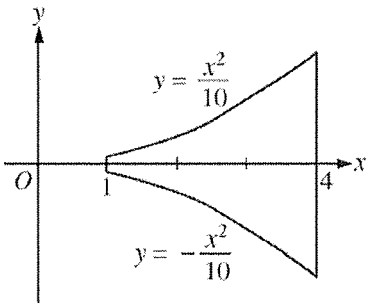


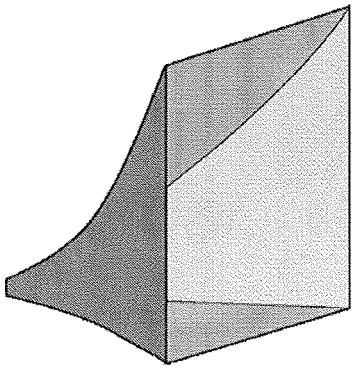

85. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$, as shown in the figure above. For this loudspeaker, the cross sections perpendicular to the $x$-axis are squares. What is the volume of the loudspeaker, in cubic units?

D

(A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

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# VOLUMES OF REVOLUTION

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the $x$-axis. What is the volume of the solid generated?

C

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) $\pi$ (D) $2\pi$ (E) $\frac{8\pi}{3}$

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30. The region enclosed by the $x$-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the $x$-axis. What is the volume of the solid generated?

C

(A) $3\pi$ (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) $9\pi$ (E) $\frac{36\sqrt{3}}{5}\pi$

---

30. What is the volume of the solid generated by rotating about the $x$-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

(A) $\frac{\pi}{\sqrt{3}}$

(B) $\pi$

(C) $\pi\sqrt{3}$

(D) $\frac{8\pi}{3}$

(E) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

C

---

36. Let $R$ be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. The volume of the solid obtained by revolving $R$ about the $x$-axis is given by

(A) $2\pi \int_0^{\frac{\pi}{2}} x \sin x \, dx$ (B) $2\pi \int_0^{\frac{\pi}{2}} x \cos x \, dx$ (C) $\pi \int_0^{\frac{\pi}{2}} (1 - \sin x)^2 \, dx$

(D) $\pi \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ (E) $\pi \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right) dx$

E

---

43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the $x$-axis is

B

(A) $2\pi$ (B) $4\pi$ (C) $6\pi$ (D) $9\pi$ (E) $12\pi$

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45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the $x$-axis is revolved about the $y$-axis. The volume of the solid generated is

A

(A) $8\pi$ (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) $4\pi$ (E) $\frac{8}{3}\pi$

---

29. The region $R$ in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when $R$ is revolved about the $y$-axis is

B

(A) $6\pi$ (B) $8\pi$ (C) $\frac{34\pi}{3}$ (D) $16\pi$ (E) $\frac{544\pi}{15}$

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23. If the region enclosed by the $y$-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the $y$-axis, the volume of the solid generated is

A

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) $\pi$

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