Section 8.2 – AP Calc MC Questions (L'Hopital's Rule)

DEFINITION OF A DERIVATIVE

6. What is $\lim_{h\to 0}\dfrac{8\left(\frac{1}{2}+h\right)^8-8\left(\frac{1}{2}\right)^8}{h}$?

B

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

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25. If $f(x)=e^x$, which of the following is equal to $f'(e)$?

E

(A) $\lim_{h\to 0}\dfrac{e^{x+h}}{h}$ (B) $\lim_{h\to 0}\dfrac{e^{x+h}-e^e}{h}$ (C) $\lim_{h\to 0}\dfrac{e^{e+h}-e}{h}$

(D) $\lim_{h\to 0}\dfrac{e^{x+h}-1}{h}$ (E) $\lim_{h\to 0}\dfrac{e^{e+h}-e^e}{h}$

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29. $\lim_{x\to\frac{\pi}{4}}\dfrac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$ is

D

(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\pi}{4}$ (D) 1 (E) nonexistent

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29. The $\lim_{h\to 0}\dfrac{\tan 3(x+h)-\tan 3x}{h}$ is

B

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

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10. $\lim_{h\to 0}\dfrac{\sin(x+h)-\sin x}{h}$ is

D

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

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14. $\lim_{h\to 0}\dfrac{e^{(2+h)}-e^2}{h}=$

D

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$

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37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

II. $\lim_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

III. $\lim_{x\to a}\dfrac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

C

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78. $\lim_{h\to 0}\dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$

(B) $f'(e)$, where $f(x)=\dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$

(E) $f'(0)$, where $f(x)=\ln x$

A

L'HOPITAL'S RULE

28. What is $\lim_{x\to 0}\dfrac{e^{2x}-1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

D

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37. $\lim_{x\to 0}\dfrac{1-\cos^2(2x)}{x^2}=$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

E

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23. If $f'(x)=\cos x$ and $g'(x)=1$ for all x, and if $f(0)=g(0)=0$, then $\lim_{x\to 0}\dfrac{f(x)}{g(x)}$ is

(A) $\dfrac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

B

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35. If k is a positive integer, then $\lim_{x\to+\infty}\dfrac{x^k}{e^x}$ is

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

A

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29. $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$ (D) 1 (E) nonexistent

C

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2. If $f(x)=2x^2+1$, then $\lim_{x\to 0}\dfrac{f(x)-f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

C

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5. $\lim_{x\to 0}\dfrac{5x^4+8x^2}{3x^4-16x^2}$ is

(A) $-\dfrac{1}{2}$ (B) 0 (C) 1 (D) $\dfrac{5}{3}+1$ (E) nonexistent

A

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2. $\lim_{x\to 0}\dfrac{2x^6+6x^3}{4x^5+3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

D

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24. Let f and g be functions that are differentiable for all real numbers, with $g(x)\neq 0$ for $x\neq 0$.
If $\lim_{x\to 0} f(x)=\lim_{x\to 0} g(x)=0$ and $\lim_{x\to 0}\dfrac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0}\dfrac{f(x)}{g(x)}$ is

(A) 0

(B) $\dfrac{f'(x)}{g'(x)}$

(C) $\lim_{x\to 0}\dfrac{f'(x)}{g'(x)}$

(D) $\dfrac{f'(x)g(x)-f(x)g'(x)}{(f(x))^2}$

(E) nonexistent

C

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16. $\lim_{h\to 0}\dfrac{e^h-1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

B

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83. If $a\neq 0$, then $\lim_{x\to a}\dfrac{x^2-a^2}{x^4-a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

B

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28. $\lim_{x\to 1}\dfrac{\int_1^x e^{t^2}\,dt}{x^2-1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

C

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23. $\lim_{h\to 0} \dfrac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$ is

C

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

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23. $\lim_{h\to 0} \dfrac{1}{h} \ln\left(\dfrac{2+h}{2}\right)$ is

C

(A) e^2 (B) 1 (C) $\dfrac{1}{2}$ (D) 0 (E) nonexistent

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37. $\lim_{x\to 0} (x \csc x)$ is

D

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞
