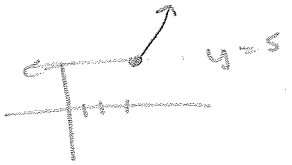
Skip 10-12-20 Key

Chapter Test Version A-A

True/False

1. If $\lim_{x \to c} f(x) = 1/2$, then $f(c) = 1/2$. False doesn't have too

2. The function $f(x) = \begin{cases} 5, & \text{if } x \le 3 \\ x^2 - 4, & \text{if } x > 3 \end{cases}$ is continuous. True



3. The function $f(x) = \dfrac{5x^2 - 3x + 2}{x + 3}$ ($(5x+2)(x-1)$) is continuous. False, V.A. at $x = -3$

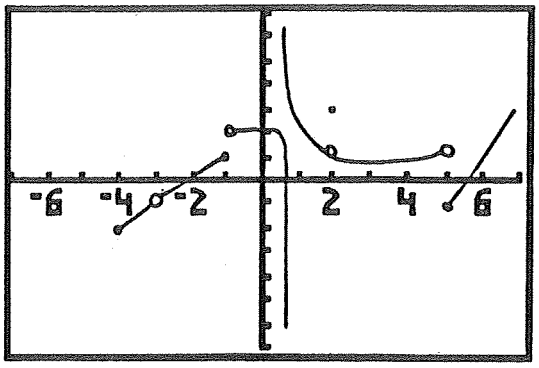
4. The function $f(x) = \dfrac{2x^2 + 17x + 35}{(x-5)(x^2 + 3x - 10)}$ ($(2x+7)(x+5)$, $(x+5)(x-2)$) has three vertical asymptotes. False

2 VA, 1 hole

5. The end behavior of $f(x) = 4x^3 + 3x^2 - 7x + 4$ and $g(x) = 4x^3$ are the same and g(x) is an end behavior model of f(x). True

Multiple Choice/Free Response

1. Consider the function $y = f(x)$ given to the right.



[-7, 7] by [-7, 7]

Which of the following statements are true for the function $y = f(x)$?

a. $f(-1) = +1$ (T); $\lim_{x \to -1} f(x) = +1$ (F); $f(x)$ is continuous at $x = -1$. (F)

(b.) $f(-4) = -2$ (T); $\lim_{x \to -4} f(x) = -2$ (T); $f(x)$ is continuous at $x = -4$. (T)

c. $f(2) = +3$ (T); $\lim_{x \to 2} f(x) = +1$ (T); $f(x)$ is continuous at $x = 2$. (F)

(d.) $f(5) = -1$ (T); $\lim_{x \to 5} f(x)$ does not exist (T); $f(x)$ is not continuous at $x = 5$. (T)

b, d is/are true.

2. Which of the following functions is/are continuous at $x = 3$?

a. $f(x) = \dfrac{x^2 + 4x + 3}{x^2 - 9}$ $\dfrac{(x+3)(x+1)}{(x-3)(x+1)}$ No

b. $\dfrac{|x + 4|}{x + 3}$ Yes

c. $f(x) = \dfrac{x^2}{x^2 + 10x + 21}$ $\dfrac{(x+7)(x+3)}{}$ Yes

d. $f(x) = \begin{cases} 3x + 2, & \text{if } x < 0 \\ (x - 1)^2, & \text{if } 0 \le x < 3 \\ x - 2, & \text{if } x \ge 3 \end{cases}$ $(3-1)^2 = 4$ $3 - 2 = 1$ No

3. Find $\lim\limits_{x \to 0} \dfrac{3 \sin x - 5 \cos x}{3x}$ $= \dfrac{3\sin 0 - 5\cos 0}{3(0)} = \dfrac{0 - 5}{0} = \dfrac{-5}{0}$

a. - 1.66667 b. 2/3 c. 5 d. does not exist

4. $f(x)$ and $g(x)$ are defined for all x and:
$\lim\limits_{x \to c} f(x) = 9$ and $\lim\limits_{x \to c} g(x) = -4$

Which of the following statement(s) is/are true?

a. $\lim\limits_{x \to c} 4 f(x) \cdot g(x) = -14$ $4(9)(-4) = -144$

b. $\lim\limits_{x \to c} (1/2) f(x) = 18$ $\frac{1}{2} \cdot 9 = 4.5$ False

c. $\lim\limits_{x \to c} f(x) - g(x) = 5$ $9 - -4 = 13$ False

d. $\lim\limits_{x \to c} \dfrac{f(x) + g(x)}{g(x)} = \dfrac{-5}{4}$ $\frac{f}{g} + 1 = \frac{-9}{4} + \frac{4}{4} = \frac{-5}{4}$ True

e. $\lim\limits_{x \to c} \dfrac{2 - g(x)}{f(x)} = \dfrac{-2}{9}$ $\frac{2}{9} \cdot \frac{-4}{9} = \frac{6}{9} = \frac{2}{3}$ False

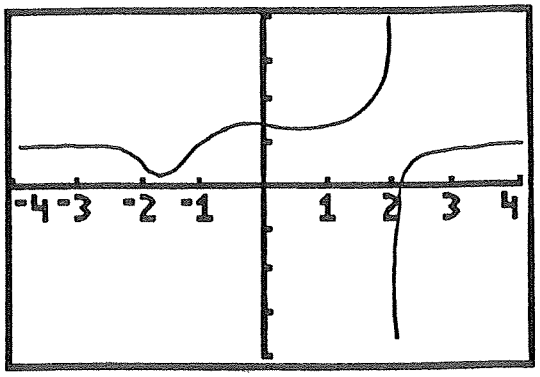
5. Consider the function $f(x)$ given to the right. Which of the following appear to be true about $f(x)$?

a. The horizontal asymptote is $y = 1$. T

b. $\lim\limits_{x \to 2} f(x) = 2$ F

c. The vertical asymptote is $x = 2$. T

d. $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to -\infty} f(x)$ T



[-4, 4] by [-4, 4]

A. a and c B. a, c, and d C. a, b, and d
D. all of the above E. none of the above

6. Find the end behavior asymptote of: $f(x) = \dfrac{x^4 - 20x^2 - 10x - 50}{x - 5}$

a. $x^3 + 5x^2 + 5x$

b. $x^3 + 5x^2 + 5x + 15$

c. $x^3 + 5x$

d. $x^3 - 5x^2 + 5x - 35$

$$x-5 \overline{)\, x^4 + 0x^3 - 20x^2 - 10x - 50}$$

$x^3 + 5x^2 + 5x + 15$

$(-)\ x^4 - 5x^3$

$5x^3 - 20x^2$

$(-)\ 5x^3 - 25x^2$

$5x^2 - 10x$

$(-)\ 5x^2 - 25x$

$15x - 50$

$15x - 75$

Use the following three functions and a--d given below to answer questions 7, 8, and 9 below.

$f(x) = \frac{5x^3 - 4x^2}{4x^3 - 7}$ $g(x) = \frac{5x^3 - 4x^2}{4x^4 - 7x}$ $k(x) = \frac{7x^2 + 3}{x - 4}$

a. 0 b. 1 c. 1.25 d. does not exist

7. The $\lim_{x \to -\infty} f(x)$ is C. $n=m$ $\frac{5x^3}{4x^3} = 1.25$

8. The $\lim_{x \to +\infty} g(x)$ is a. $n<m$ $y=0$

9. The $\lim_{x \to +\infty} k(x)$ is d. $n>m$ No H.A. $k(100) = \frac{7(100)^2+3}{100-4} = \frac{+}{+} = \infty$

Select answers for questions 10 and 11 from the following:

a. $|x - 1| < 6$ b. $-8 < x - 3 < 4$ c. $|x - 4| < 6$ d. $|x - 6| < 4$.

10. Describe the interval $-2 < x < 10$ in the form: $|x - x_0| < D$ C

11. Which of the above statements is/are true for $-5 < x < 7$? a, b

Skip

12. In what interval must x_0 be held to be sure that $y = f(x)$ lies within 0.1 unit of $y_0 = 1.5$, where $y = f(x) = \frac{2x - 3}{x - 2}$

a. $-.3333 < x < 5$ b. $-.3333 < x < .5$
c. $.3333 < x < .5$ (d.) $-.5 < x < .3333$

Use the given functions a - e to answer questions 13, 14, and 15 below.

a. $v(x) = \frac{x(x+2)(x-2)}{x+2}$ hole at $x=-2$ b. $k(x) = \frac{(1/2)x}{x+2}$ VA $x=-2$

c. $m(x) = \frac{3x+2}{x+2}$ VA at $x=-2$ d. $p(x) = \frac{x^2 - 5x - 14}{x+3}$

e. $t(x) = x^3 - 2$

13. For which of these functions is $f(-2)$ defined? d, e.

14. For which of these functions does the limit as $x \to -2$ exist? a, d, e.

15. For which of these functions does the $\lim_{x \to -2} f(x) = f(-2)$? d, e.

16. For what value of k in the function given below does the $\lim_{x \to 2} f(x)$ exist? $k = 1$

$$f(x) = \begin{cases} x^2 - 2x - 1, & \text{if } x \le 2 \\ -x + k, & \text{if } x > 2. \end{cases}$$

$2^2 - 2(2) - 1 = 4 - 4 - 1 = -1$

$-x + k = -1$
$-2 + k = -1$
$+2 \quad +2$
$k = 1$

17. Imperial College of Medicine wants to remove impurities from waste water in one of its research labs. The cost C of removing p percent of the impurities from the water is given by:

$$C(p) = \frac{7600p}{100 - p}$$

$p = 100$ V.A.

Describe any discontinuities for C(p), if they exist. In your opinion, about what percentage [you can select a percent interval] of the impurities should be removed from the waste water?

Function not continuous at 100%. Money/budget constraints as possible restrictions on what percentage of the impurities the college chooses to remove.

18. The function f(x) is defined by: $f(x) = \begin{cases} x^2 + 4x - 2, & \text{if } x < -2 \\ x + k, & \text{if } x \geq -2. \end{cases}$

$(-2)^2 + 4(-2) - 2 = 4 - 8 - 2 = -6$

$-2 + k = -6$

$k = -4$

What value must k be equal to such that f(x) is continuous at $x = -2$?

$k = -4$

19. $f(x) = \dfrac{2x + \sin x}{2x}$

Estimate the $\lim_{x \to 0} f(x)$ graphically. 1.5

Prove it algebraically.

$$= \lim_{x \to 0} \left(\frac{2x}{2x} + \frac{1}{2} \frac{\sin x}{x} \right)$$

$$= \lim_{x \to 0} 1 + \frac{1}{2} \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1 + \frac{1}{2} \cdot 1$$

$$= 1.5$$

~~20~~. skip $f(x) = 2x + 7$, $x_0 = 5$, $\varepsilon = 0.01$. Find $L = \lim_{x \to x_0} f(x)$.

Then find a number $\delta > 0$ such that for all x,

$0 < |x - x_0| < \delta$ -------> $|f(x) - L| < \varepsilon$

.005