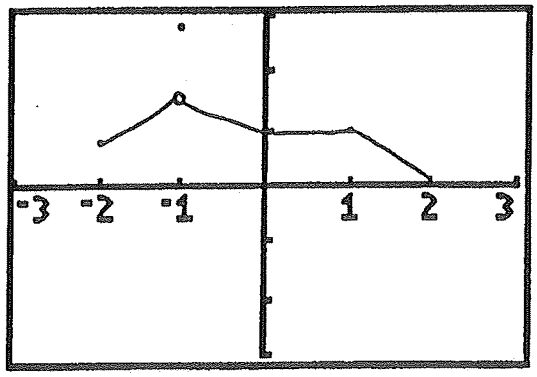
Skip 5,10-12, 20 Key

Chapter Test Version A-B

True/False

1. The $\lim_{x \to c} f(x)$ can exist even if $f(c)$ is undefined. True

2. The function given below is continuous on the interval [-2, 2]. False (hole at $x=-1$)



[-3, 3] by [-3, 3].

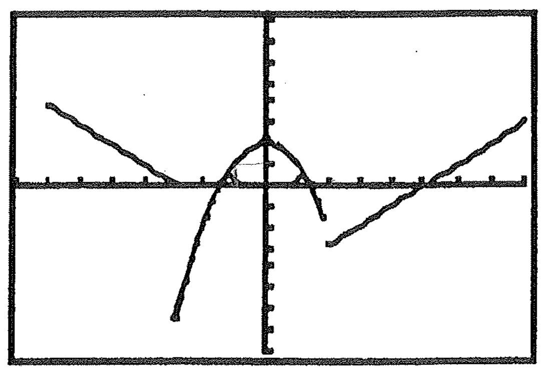
3. If $f(x)$ is a polynomial function, then $\lim_{x \to c} f(x) = f(c)$ always. True — A polynomial's domain is all Reals

4. If $y = f(x)$ lies within one unit of $y = -3$, then $-4 < y < -2$. True

Skip ~~5.~~ In proving $\lim_{x \to 1} (5x - 3) = 2$, we find that $\delta = \varepsilon/5$. True

Multiple Choice/Free Response

1. Consider the function $y = f(x)$ given to the right.



[- 8, 8] by [- 8, 8]

Which of the following statements are true for the function $y = f(x)$?

a. $f(-1) \overset{T}{=} +1$; $\lim_{x \to -1} f(x) \overset{T}{=} +1$; $f(x)$ is continuous at $x = -1$. T

b. $f(-3) \overset{T}{=} 0$; $\lim_{x \to -3} f(x) \overset{F}{=} 0$; $f(x)$ is continuous at $x = -4$. T

c. $f(2) \overset{F}{=} +2$; $\lim_{x \to 2} f(x) \overset{F}{=} +2$; $f(x)$ is continuous at $x = 2$. F

d. $f(5) \overset{F}{=} 1$; $\lim_{x \to 5} f(x)$ does not exist F ; $f(x)$ is not continuous at $x = 5$. F

2. Which of the following functions is/are continuous at $x = 5$?

a. $f(x) = \dfrac{x^2 - 11x + 30}{x^2 - 25}$ No $(x+5)(x-5)$

b. $\dfrac{|x+2|}{x-7}$ yes

c. $f(x) = \dfrac{x^2}{x^2 + 10x + 25}$ yes $(x+5)(x+5)$

d. $f(x) = \begin{cases} 3x + 2, & \text{if } x < 0 \\ (x-1)^2, & \text{if } 0 \le x < 5 \\ x - 2, & \text{if } x \ge 5 \end{cases}$ $(5-1)^2 = 4^2 = 16$ $5-2=3$ jump

3. Find $\lim_{x \to 0} \dfrac{2 \sin 3x - 7 \cos x}{3x}$ $= \dfrac{2\cdot 0 - 7 \cdot 1}{0} = \dfrac{-7}{0}$ = DNE or $\pm\infty$

a. - 1.66667 b. 2/3 c. 5.372471 d. does not exist

4. f(x) and g(x) are defined for all x and:
$\lim_{x \to c} f(x) = 15$ and $\lim_{x \to c} g(x) = -3$

Which of the following statement(s) is/are true?

a. $\lim_{x \to c} 4 f(x) \cdot g(x) = -14$ No $4 \cdot 15 \cdot -3$

b. $\lim_{x \to c} (1/2) f(x) = 7.5$ $\frac{1}{2} \cdot 15 = 7.5$ yes

c. $\lim_{x \to c} f(x) - g(x) = 18$ $15 - -3 = 18$ yes

d. $\lim_{x \to c} \dfrac{f(x) + g(x)}{g(x)} = -6$ No $\frac{f}{g} + 1 = \frac{15}{-3} + 1$ $-5+1=-4$

e. $\lim_{x \to c} \dfrac{2 - g(x)}{f(x)} = \dfrac{-1}{3}$ NO $\frac{2}{f(x)} - \frac{g}{f} = \frac{2}{15} - \frac{-3}{15} = \frac{5}{15} = \frac{1}{3}$

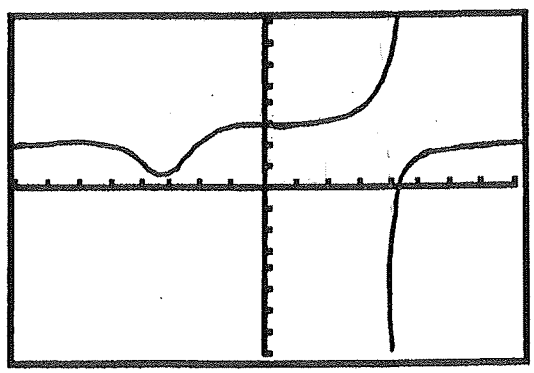
5. Consider the function f(x) given to the right. Which of the following appear to be true about f(x)?

a. The horizontal asymptote is y = 2. True

b. $\lim_{x \to 2} f(x) = \cancel{2}$ 3 False

c. The vertical asymptote is x = 4. True a little off...

d. $\lim_{x \to +\infty} f(x) = \lim_{x \to -\infty} f(x)$ True



[- 8, 8] by [- 8, 8]

A. a , c, and d B. a and c C. a, b, and d
D. all of the above E. none of the above

6. Find the end behavior asymptote of: $f(x) = \dfrac{x^4 + 3x^3 - 2x^2 - 6x + 7}{x + 2}$

a. $x^3 + x^2 + 4x$ b. $x^3 + x^2 - 4x + 2$

c. $x^3 + 2x$ d. $x^3 - 2x^2 + 4x - 35$

$$\begin{array}{l} \quad x^3 + x^2 - 4x + 2 \\ x+2 \,\big)\, x^4 + 3x^3 - 2x^2 - 6x + 7 \\ (-)\ x^4 + 2x^3 \\ \qquad x^3 - 2x^2 \\ (-)\ x^3 + 2x^2 \\ \qquad -4x^2 - 6x \\ (-)\ -4x^2 - 8x \\ \qquad\qquad 2x + 7 \\ (-)\ \quad 2x + 4 \\ \qquad\qquad\quad 3 \end{array}$$

Use the following three functions and a--d given below to answer questions 7, 8, and 9 below.

$f(x) = \dfrac{5x^5 - 3x^2}{7x^3 - 8}$ $g(x) = \dfrac{5x^3 - 4x^2}{4x^3 - 7x}$ $k(x) = \dfrac{5x^2 + 3}{x^3 - 6}$

a. 0 b. 1.25 c. 1.50 d. does not exist

7. The $\lim_{x \to -\infty} f(x)$ is d. $n > m$, it's going to ∞

8. The $\lim_{x \to +\infty} g(x)$ is b. $n = m$, $y = \frac{5}{4}$

9. The $\lim_{x \to +\infty} k(x)$ is a. $n < m$, $y = 0$

Select answers for questions 10 and 11 from the following:

a. $|x - 3| < 7$ b. $-10 < x - 4 < 7$ c. $|x - 4| < 6$ d. $|x - 5| < 8$.

10. Describe the interval $-3 < x < 13$ in the form: $|x - x_0| < D$ d

11. Which of the above statements is/are true for $-5 < x < 7$? b

12. In what interval must x_0 be held to be sure that $y = f(x_0)$ lies within 0.1 unit of $y_0 = 1.5$, where $y = f(x_0) = \dfrac{3x + 4}{x + 3}$ c

a. $-2/7 < x_0 < -1/8$ b. $-1/8 < x_0 < 2/7$
c. $1/8 < x_0 < 2/7$ d. $-2/7 < x_0 < 1/8$

Use the given functions a - e to answer questions 13, 14, and 15 below.

a. $v(x) = \dfrac{x\cancel{(x+5)}(x-5)}{\cancel{x+5}}$ hole: $x = -5$

b. $m(x) = \dfrac{3x + 5}{x + 5}$ VA: $x = -5$

c. $t(x) = x^3 - 7$

d. $k(x) = \dfrac{(1/2)x}{x + 5}$ VA: $x = -5$

e. $p(x) = \dfrac{x^2 - 5x - 14}{x + 3}$ $(x-7)(x+2)$ VA: $x = -3$

13. Which of these functions is/are defined at $x = -5$? c, e.

14. For which of these functions does the limit as $x \to -5$ exist ? a, c, e.

15. Which of these functions is/are continuous for $x = -5$) ? c, e.

16. For what value of k in the function given below does the $\lim_{x \to 3} f(x)$ exist? $K = -3$

$$f(x) = \begin{cases} x^2 - 5x - 3, & \text{if } x \le 3 \\ -2x + k, & \text{if } x > 3. \end{cases}$$

$3^2 - 5(3) - 3 = 9 - 15 - 3 = -9$

$-2(3) + K = -9$

$K = -3$

17. New London University wants to remove impurities from waste water in one of its research labs. The cost C of removing p percent of the impurities from the water is given by:

$$C(p) = \frac{8300p}{87 - p}$$

$87 - p = 0$

$87 = p$

Describe any discontinuities for C(p), if they exist. In your opinion, about what percentage [you can select a percent interval] of the impurities should be removed from the waste water?

Not continuous at 87%

Money/budget constraints as possible restrictions on what % of the impurities the college chooses to remove.

18. The function f(x) is defined by: $f(x) = \begin{cases} x^2 - 6x - 2, & \text{if } x < -1 \\ x + k, & \text{if } x \geq -1. \end{cases}$

What value must k equal in order that f(x) is continuous at x = - 1?

$K = 6$

19. $f(x) = \dfrac{2x + \sin 2x}{x}$ $= \frac{0}{0}$ undeterminate

Estimate the $\lim_{x \to 0} f(x)$ graphically. +4

Prove it algebraically.

~~20~~. $f(x) = 3x - 5$, $x_0 = 7$, $\varepsilon = 0.001$. Find $L = \lim_{x \to x_0} f(x)$.

Then find a number $\delta > 0$ such that for all x,

$.0003333$

$0 < | x - x_0 | < \delta \quad \text{------->} \quad | f(x) - L | < \varepsilon$

$x^2 - 6x - 2$

$(-1)^2 - 6(-1) - 2 = 1 + 6 - 2$
$= 5$

$x + k = 5$

$-1 + k = 5$

$k = 6$

$-1 \leq \sin x \leq 1$

$-1 \leq \sin(2x) \leq 1$

$2x + -1 \leq 2x + \sin 2x \leq 1 + 2x$

$$\frac{2x + -1}{x} \leq \frac{2x + \sin 2x}{x} \leq \frac{1 + 2x}{x}$$

$$\lim_{x \to 0} \frac{2x + -1}{x} \leq \lim_{x \to 0} \frac{2x + \sin 2x}{x} \leq \lim_{x \to 0} \frac{1 + 2x}{x}$$

$-\infty \leq \quad \leq \infty$

doesn't work since different

$\rightarrow \lim_{x \to 0} \frac{2x}{x} + \frac{\sin 2x}{x}$

identity $\sin 2x = 2 \sin x \cos x$

$\lim_{x \to 0} 2 + \frac{2 \sin x \cos x}{x} =$

$= \lim_{x \to 0} 2 + 2\cos x \cdot \left(\frac{\sin x}{x}\right)$

$= 2 + 2\cos 0 * 1 = 2 + 2 = 4$