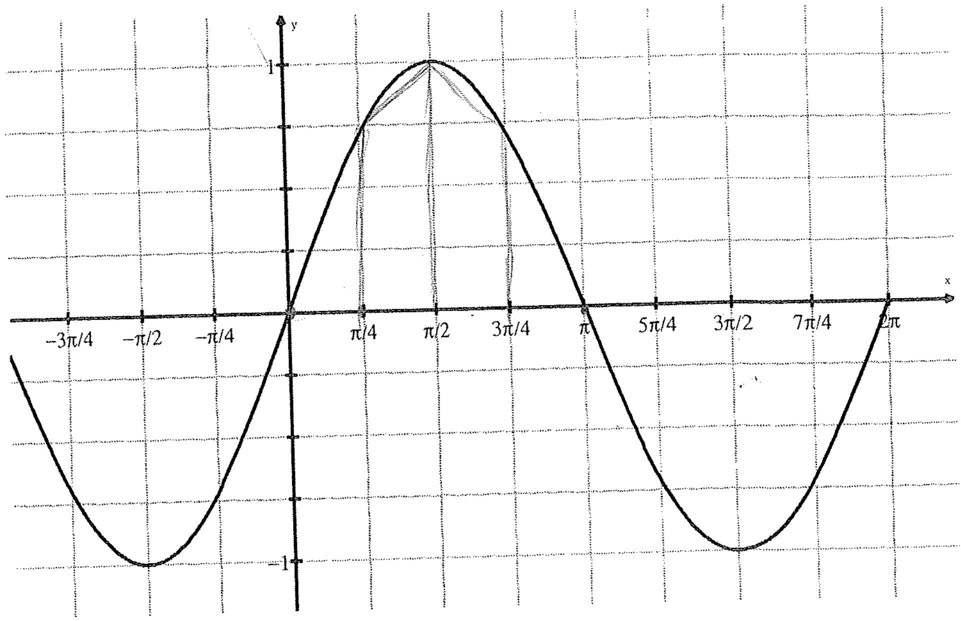
* Can use after 5-2 ... even some trapezoidal questions are on it.

Key

AP Calculus
A Return To Riemann Sums

Review of Riemann Sums

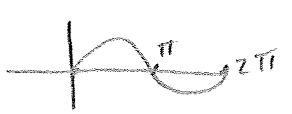
Last week we reviewed the idea of estimating the area under a curve. There are four popular methods known as Riemann Sums: the left-end-point method, the right-endpoint method, the midpoint method, the trapezoidal rule. For the graph of the function $f(x) = \sin(x)$ on the interval $[0, \pi]$, estimate the area bounded by the curve and the x-axis using the following methods.



	Indicated Method	*The Set Up*	*The Final Answer*
1	4 rectangles of equal width and the left endpoint method	$= \frac{\pi}{4}\left(0 + \sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4}\right)$ $= \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}\left(\frac{2\sqrt{2}+2}{2}\right)$	$\frac{\pi}{4}(\sqrt{2}+1) = \boxed{\frac{\pi\sqrt{2}}{4} + \frac{\pi}{4}} = \boxed{1.896}$
2	4 rectangles of equal width and the right endpoint method	$= \frac{\pi}{4}\left(\sin\pi/4 + \sin\frac{\pi}{2} + \sin 3\pi/4 + \sin\pi\right)$ $= \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0\right)$	$\frac{\pi}{4}(\sqrt{2}+1) = \boxed{\frac{\pi\sqrt{2}}{4} + \frac{\pi}{4}} = \boxed{1.896}$
3	4 rectangles of equal width and the midpoint method	$= \frac{\pi}{4}\left(\sin\frac{\pi}{8} + \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} + \sin\frac{7\pi}{8}\right)$	$= \boxed{2.052}$
4	4 trapezoids of equal width (height) $h = \frac{\pi - 0}{4} = \frac{\pi}{4}$ $\frac{\pi}{4} \cdot \frac{1}{2}$	$A = \frac{\pi}{8}\left(0 + 2\cdot\sin\frac{\pi}{4} + 2\cdot\sin\frac{\pi}{2} + 2\cdot\sin\frac{3\pi}{4} + 0\right)$ $= \frac{\pi}{8}\left(\cancel{2}\frac{\sqrt{2}}{\cancel{2}} + 2\cdot 1 + \cancel{2}\frac{\sqrt{2}}{\cancel{2}}\right) = \frac{\pi}{8}(2\sqrt{2}+2)$	$= \boxed{1.896}$
5	Using the definite integral, find the exact area.	$\int_0^\pi \sin x\,dx = -\cos x\Big\|_0^\pi =$ $-\cos\pi + \cos 0 =$	$1 + 1 = 2$ 2

Extension Question

If you use your calculator, you will discover that $\int_0^{2\pi} \sin(x)\,dx = 0$. Explain why this is true.



The area above & below the x-axis is equal & "cancel" eachother out.

AP Calculus
A Return To Riemann Sums

Connection To Physics

We previously connected the ideas of position, velocity and acceleration together using the derivative. There are more connections that we will be exploring over the coming weeks.

Sample Question 1

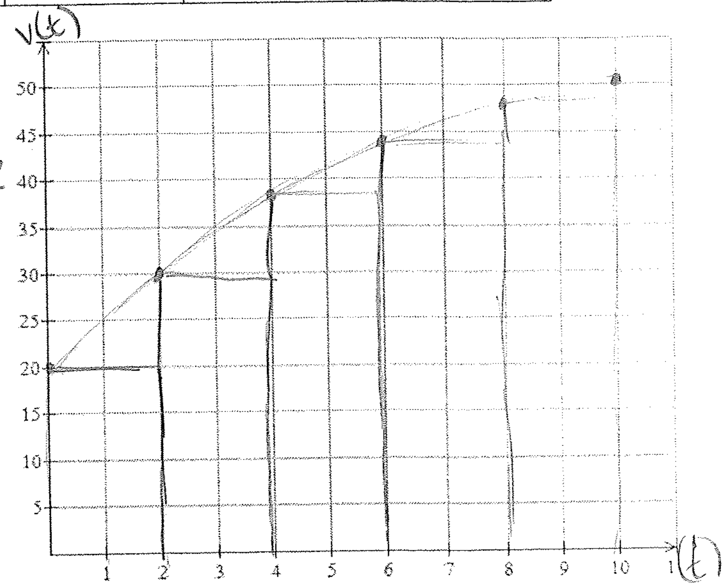
A car is moving with **increasing** velocity. The table below shows the velocity every 2 seconds.

Time (sec)	0	2	4	6	8	10
Velocity (m/sec)	20	30	38	44	48	50

(a) How can we estimate the distance traveled by this car in 10 seconds? Be sure to indicate whether your estimate will be an overestimate or an under estimate.

LRAM – will be an underestimate

$2(20+30+38+44+48) = \boxed{360 \text{ meters}}$



(b) Now, what if we had more data as in the table below? How can this additional information help us in obtaining a more accurate estimate of the distance traveled? As the number of rectangles increase, the closer to the actual area we will be. The area between the curve + rectangle will be less + it will not be as much as an under-estimate

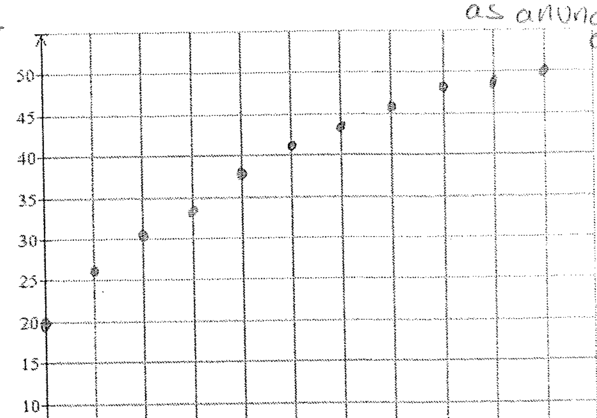
Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/sec)	20	26	30	34	38	41	44	46	48	49	50

(c) Find the lower and upper estimates for your distance. Create a scatter plot for your data to visualize the idea.

LRAM $\rightarrow 1(20+26+30+34+38+41+44+46+48+49) = \boxed{420}$

R-RAM $\rightarrow 1(26+30+34+38+41+44+46+48+49+50)$

$= \boxed{450}$



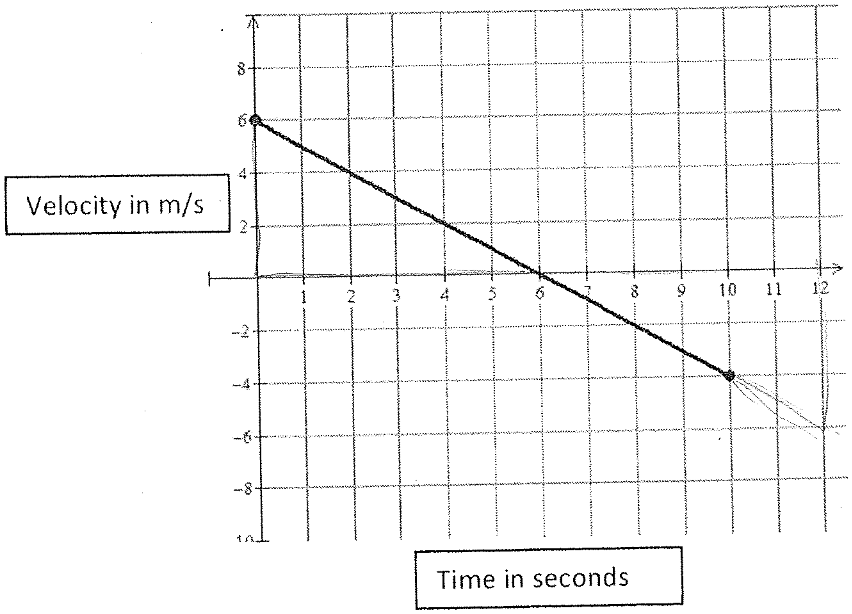
AP Calculus
A Return To Riemann Sums

Sample Question 2

What does negative velocity mean? the object turned direction

Let's look at an example of area under a curve in which the velocity is negative.

1. A particle starts 2 meters to the right of the origin and moves horizontally with velocity according to the graph below.



Distance - how much ground an object has covered

Displacement - how far out of place an object is
final position − initial position
"Change in position"

2 16 18
10 8

a) What is the distance traveled by the particle after 6 seconds? $\frac{1}{2}\cdot 6\cdot 6 = \boxed{18m}$

b) What is the displacement of the particle after 6 seconds? What is the position of the particle after 6 seconds?

final position − initial position
$18 - 2 = \boxed{16}$

$2 + 18 = $ 20 m to right of origin

c) What is the distance traveled by the particle at 10 seconds?

$\frac{1}{2}\cdot 4\cdot 4 = 8$

$18 + 8 = 26m$

d) What is the displacement of the particle at 10 seconds? What is the position of the particle at 10 seconds?

2 18 20
12 8

10

2 16 18
10 8

12 m right of origin

e) If the particle were to continue with velocity given by the same linear function after 10 seconds, at what time will the particle return to its starting point? What is the total displacement of the particle at that point? What is the total distance traveled?

$2 - 2 = \boxed{0}$

Equal areas at $t=12$

$\boxed{t = 12s}$

$18 + 18 = \boxed{36m}$

AP Calculus
A Return To Riemann Sums
Sample AP Questions

2003 BC Exam Question #25

x	2	5	10	14
$f(x)$	12	28	34	30

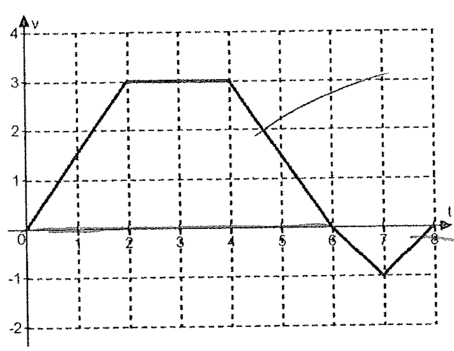
The function f is continuous on the closed interval $[2,14]$ and has values as shown in the table above. Using the subintervals $[2,5]$, $[5,10]$, and $[10,14]$, what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

$= 3 \cdot 28 + 5 \cdot 34 + 4 \cdot 30$

(A) 296 (B) 312 (C) 343

(D) 374 (E) 390

1997 AB Exam Question #9



$A = \frac{1}{2} \cdot 3(2+6) = 12$

$A = \frac{1}{2} 2 \cdot 1 = 1$

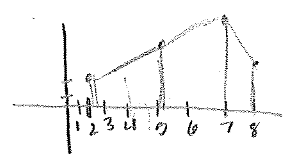
A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le x \le 8$, is given by the function whose graph is shown above. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

$12 + 1 = 13$

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

1998 AB Exam Question #85

x	2	5	7	8
f(x)	10	30	40	20



The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, $[7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$

not equal

(A) 110 (B) 130 (C) 160

(D) 190 (E) 210

$\frac{1}{2} 3 \cdot (10+30) + \frac{1}{2} \cdot 2(30+40) + \frac{1}{2} \cdot 1(40+20)$

$= 60 + 70 + 30$

$= 160$