Section 2.1 Exercises

1. $\dfrac{\Delta y}{\Delta t} = \dfrac{16(3)^2 - 16(0)^2}{3-0} = 48\ \text{ft/sec}$

3. $\dfrac{\Delta y}{\Delta t} = \dfrac{16(3+h)^2 - 16(3)^2}{h}$, say $h = 0.01$

$$= \frac{16(3+0.01)^2 - 16(9)}{0.01} = \frac{16(9.0601) - 16(9)}{0.01}$$

$$= \frac{144.9616 - 144}{0.01} = \frac{0.9616}{0.01} = 96.16\ \text{ft/sec}$$

Confirm Algebraically

$$\frac{\Delta y}{\Delta t} = \frac{16(3+h)^2 - 16(3)^2}{h}$$

$$= \frac{16(9+6h+h^2) - 144}{h} = \frac{96h + 16h^2}{h} = (96 + 16h)\ \text{ft/sec}$$

if $h = 0$, then $\dfrac{\Delta y}{\Delta t} = 96\dfrac{\text{ft}}{\text{sec}}$

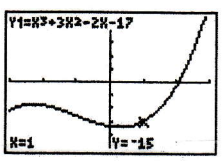
5. $\lim\limits_{x \to c} (2x^3 - 3x^2 + x - 1)$

$$= 2c^3 - 3c^2 + c - 1$$

9. $\lim\limits_{x \to 1} (x^3 + 3x^2 - 2x - 17) = (1)^3 + 3(1)^2 - 2(1) - 17$

$$= 1 + 3 - 2 - 17 = -15$$

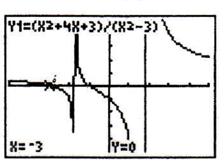
Graphical support:



[−3, 3] by [−25, 25]

11. $\lim\limits_{y \to -3} \dfrac{y^2 + 4y + 3}{y^2 - 3} = \dfrac{(-3)^2 + 4(-3) + 3}{(-3)^2 - 3} = \dfrac{0}{6} = 0$

Graphical support:

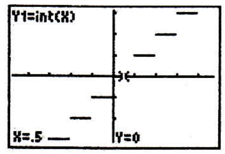


[−5, 5] by [−5, 5]

12. $\lim\limits_{x \to 1/2} \operatorname{int} x = \operatorname{int} \dfrac{1}{2} = 0$

Note that substitution cannot always be used to find limits of the int function. Its use here can be justified by the Sandwich Theorem, using $g(x) = h(x) = 0$ on the interval (0, 1).

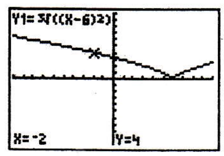
Graphical support:



[−4.7, 4.7] by [−3.1, 3.1]

13. $\lim\limits_{x \to -2} (x-6)^{2/3} = (-2-6)^{2/3} = \sqrt[3]{(-8)^2} = \sqrt[3]{64} = 4$

Graphical support:

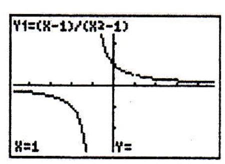


[−10, 10] by [−10, 10]

15. You cannot use substitution because the expression $\sqrt{x-2}$ is not defined at $x = -2$. Since the expression is not defined at points near $x = -2$, the limit does not exist.

17. You cannot use substitution because the expression $\dfrac{|x|}{x}$ is not defined at $x = 0$. Since $\lim\limits_{x \to 0^-} \dfrac{|x|}{x} = -1$ and $\lim\limits_{x \to 0^+} \dfrac{|x|}{x} = 1$, the left- and right-hand limits are not equal and so the limit does not exist.

19.



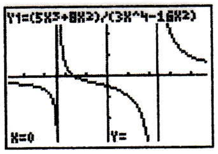
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{1}{2}$$

Algebraic confirmation:

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{x-1}{(x+1)(x-1)} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$$

21.



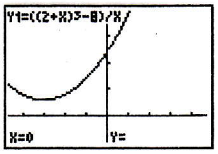
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}=-\frac{1}{2}$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}=\lim_{x\to 0}\frac{x^2(5x+8)}{x^2(3x^2-16)}$$
$$=\lim_{x\to 0}\frac{5x+8}{3x^2-16}$$
$$=\frac{5(0)+8}{3(0)^2-16}$$
$$=\frac{8}{-16}=-\frac{1}{2}$$

23.



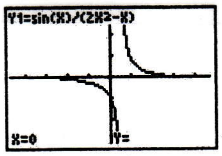
[−4.7, 4.7] by [−5, 20]

$$\lim_{x\to 0}\frac{(2+x)^3-8}{x}=12$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{(2+x)^3-8}{x}=\lim_{x\to 0}\frac{12x+6x^2+x^3}{x}$$
$$=\lim_{x\to 0}(12+6x+x^2)$$
$$=12+6(0)+(0)^2=12$$

25.



[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{\sin x}{2x^2-x}=-1$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{\sin x}{2x^2-x}=\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{2x-1}\right)$$
$$=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{2x-1}\right)=(1)\left(\frac{1}{2(0)-1}\right)=-1$$

31. Since $\operatorname{int} x=0$ for x in (0, 1), $\lim_{x\to 0^+}\operatorname{int} x=0$.

32. Since $\operatorname{int} x=-1$ for x in (−1, 0), $\lim_{x\to 0^-}\operatorname{int} x=-1$.

37. (a) True

(b) True

(c) False, since $\lim_{x\to 0^-} f(x)=0$.

(d) True, since both are equal to 0.

(e) True, since (d) is true.

(f) True

(g) False, since $\lim_{x\to 0} f(x)=0$.

(h) False, $\lim_{x\to 1^-} f(x)=1$, but $\lim_{x\to 1} f(x)$ is undefined.

(i) False, $\lim_{x\to 1^+} f(x)=0$, but $\lim_{x\to 1} f(x)$ is undefined.

(j) False, since $\lim_{x\to 2^-} f(x)=0$.

39. (a) $\lim_{x\to 3^-} f(x)=3$

(b) $\lim_{x\to 3^+} f(x)=-2$

(c) $\lim_{x\to 3} f(x)$ does not exist, because the left- and right-hand limits are not equal.

(d) $f(3)=1$

41. (a) $\lim_{h\to 0^-} f(h)=-4$

(b) $\lim_{h\to 0^+} f(h)=-4$

(c) $\lim_{h\to 0} f(h)=-4$

(d) $f(0)=-4$

43. (a) $\lim_{x\to 0^-} F(x)=4$

(b) $\lim_{x\to 0^+} F(x)=-3$

(c) $\lim_{x\to 0} F(x)$ does not exist, because the left- and right-hand limits are not equal.

(d) $F(0)=4$

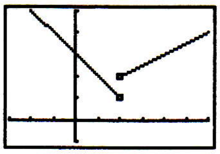
49. (a) $\lim_{x\to 4}(g(x)+3) = \left(\lim_{x\to 4} g(x)\right) + \left(\lim_{x\to 4} 3\right) = 3+3=6$

(b) $\lim_{x\to 4} x f(x) = \left(\lim_{x\to 4} x\right)\left(\lim_{x\to 4} f(x)\right) = 4\cdot 0 = 0$

(c) $\lim_{x\to 4} g^2(x) = \left(\lim_{x\to 4} g(x)\right)^2 = 3^2 = 9$

(d) $\lim_{x\to 4} \frac{g(x)}{f(x)-1} = \frac{\lim_{x\to 4} g(x)}{\left(\lim_{x\to 4} f(x)\right) - \left(\lim_{x\to 4} 1\right)} = \frac{3}{0-1} = -3$

51. (a)

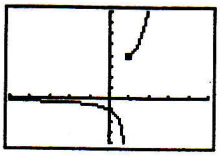


[–3, 6] by [–1, 5]

(b) $\lim_{x\to 2^+} f(x) = 2$; $\lim_{x\to 2^-} f(x) = 1$

(c) No, because the two one-sided limits are different.

53. (a)



[–5, 5] by [–4, 8]

(b) $\lim_{x\to 1^+} f(x) = 4$; $\lim_{x\to 1^-} f(x)$ does not exist.

(c) No, because the left-hand limits does not exist.