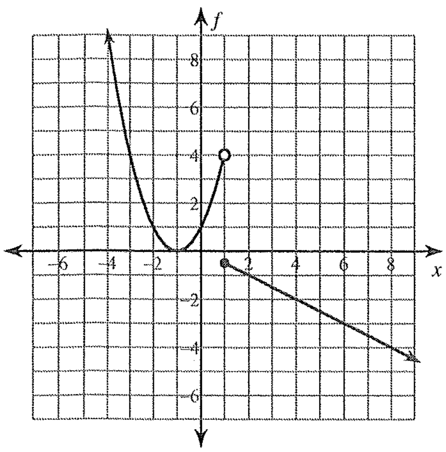
Kuta Software - Infinite Calculus *See skipped problems Name Key

Continuity

2.3 + 2.4 Review Date________________ Period____

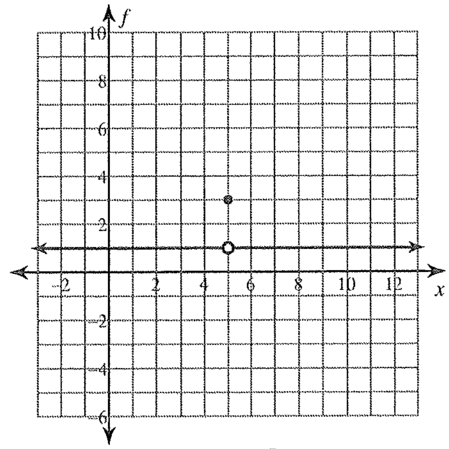
Find the intervals on which each function is continuous.

1) $f(x) = \begin{cases} x^2 + 2x + 1, & x < 1 \\ -\dfrac{x}{2}, & x \geq 1 \end{cases}$



$(-\infty, 1), [1, \infty)$

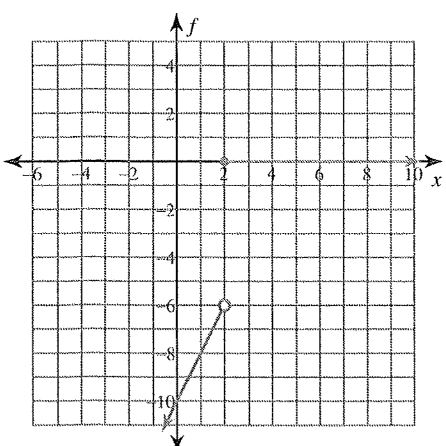
2) $f(x) = \begin{cases} 1, & x \neq 5 \\ 3, & x = 5 \end{cases}$



$(-\infty, 5), (5, \infty)$

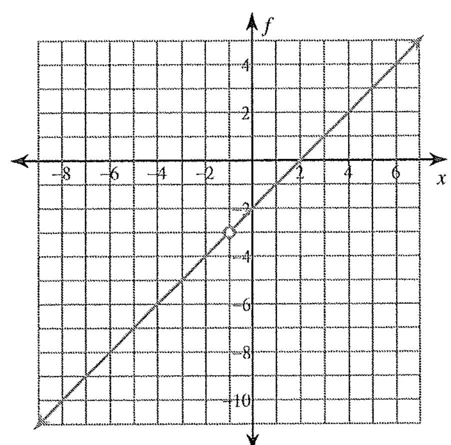
Find the intervals on which each function is continuous. You may use the provided graph to sketch the function.

3) $f(x) = \begin{cases} 2x - 10, & x < 2 \\ 0, & x \geq 2 \end{cases}$



$(-\infty, 2), [2, \infty)$

4) $f(x) = \dfrac{x^2 - x - 2}{x + 1}$



$(-\infty, -1), (-1, \infty)$

Find the intervals on which each function is continuous.

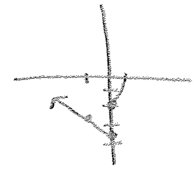
$\frac{-x-7}{2}$

5) $f(x) = \frac{x^2}{2x+4}$

$2x+4=0$
$2x=-4$
$x=-2$

$(-\infty, -2), (-2, \infty)$

6) $f(x) = \begin{cases} -\frac{x}{2} - \frac{7}{2}, & x \le 0 \\ -x^2 + 2x - 2, & x > 0 \end{cases}$



$(-\infty, 0], (0, \infty)$

7) $f(x) = -\frac{x^2 - x - 12}{x+3}$

$(x-4)(x+3)$

$(-\infty, -3), (-3, \infty)$

8) $f(x) = \frac{x^2 - x - 6}{x+2}$

$(x-3)(x+2)$

hole at $x=-2$

$(-\infty, -2), (-2, \infty)$

Determine if each function is continuous. If the function is not continuous, find the *x*-axis location of and classify each discontinuity.

9) $f(x) = -\frac{x^2}{2x+4}$

$2x+4=0$
$2x=-4$
$x=-2$
V.A.

Essential discontinuity at: $x = -2$
OR
infinite

10) $f(x) = \frac{x+1}{x^2 - x - 2}$

$(x+1)(x-2)$

hole: $x=-1$
VA: $x=2$

Removable discontinuity at: $x = -1$
Essential discontinuity at: $x = 2$
or
infinite

11) $f(x) = \frac{x+1}{x^2 + x + 1}$ ← No places where equal to 0.

Continuous

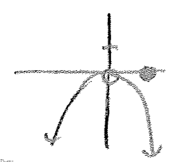
12) $f(x) = -\frac{x^2}{x-1}$

Essential discontinuity at: $x = 1$
or
infinite

13) $f(x) = \begin{cases} x^2 - 4x + 3, & x \ne 0 \\ 3, & x = 0 \end{cases}$

Continuous

14) $f(x) = \begin{cases} -x^2, & x \ne 1 \\ 0, & x = 1 \end{cases}$



Removable discontinuity at: $x = 1$

Critical thinking questions:

15) Give an example of a function with discontinuities at $x = 1$, 2, and 3.

Many answers. $\frac{1}{(x-1)(x-2)(x-3)}$

16) Of the six basic trigonometric functions, which are continuous over all real numbers? Which are not? What types of discontinuities are there?

Cont: sin, cos. Not cont: sec, csc, tan, cot. Essential.

Kuta Software - Infinite Calculus

Name Key

Average Rates of Change

Date________________ Period____

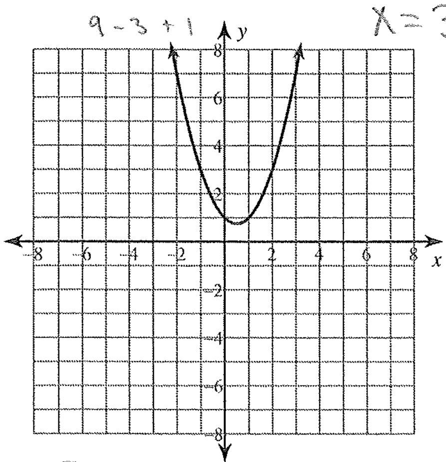
For each problem, find the average rate of change of the function over the given interval.

1) $y = x^2 - x + 1$; $[0, 3]$

$x=0 \quad y=1$

$x=3 \quad y=7$

$9-3+1$



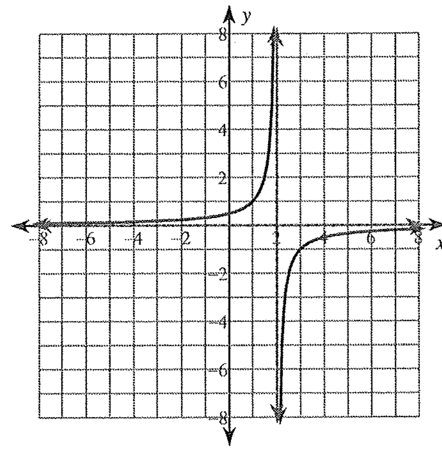
$\frac{7-1}{3-0} = \frac{6}{3} = 2$

2

2) $y = -\frac{1}{x-2}$; $[-3, -2]$

$x=-3 \quad y=\frac{1}{5}$

$x=-2 \quad y=\frac{1}{4}$



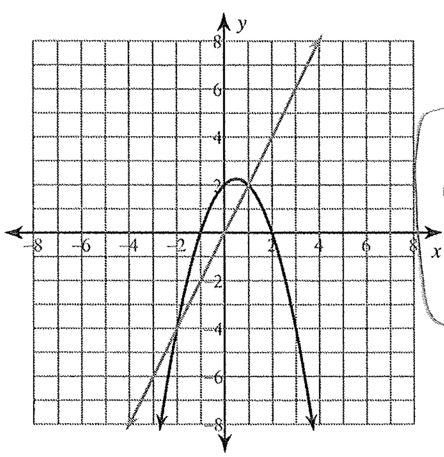
$m = \frac{\frac{1}{4}-\frac{1}{5}}{-2+3} = \frac{\frac{5}{20}-\frac{4}{20}}{1} = \frac{1}{20}$

$\frac{1}{20}$

For each problem, find the equation of the secant line that intersects the given points on the function.

3) $y = -x^2 + x + 2$; $(-2, -4)$, $(1, 2)$

$m = \frac{2+4}{1+2} = \frac{6}{3} = 2$



$y-2=2(x-1)$

or

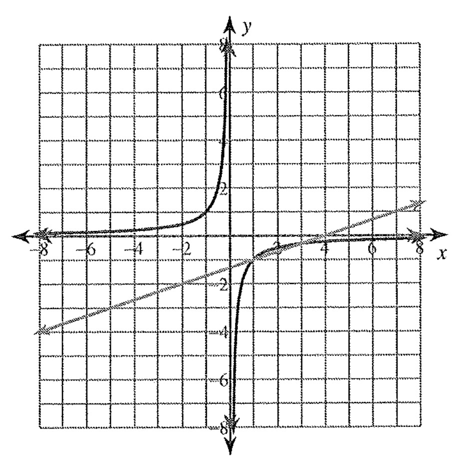
$y=2x$

$y = 2x$

4) $y = -\frac{1}{x}$; $(1, -1)$, $\left(3, -\frac{1}{3}\right)$

$m = \frac{-1+\frac{1}{3}}{1-3} = \frac{-\frac{2}{3}}{-2}$

$-\frac{2}{3} \cdot \frac{1}{-2} = \frac{1}{3}$



$y+1=\frac{1}{3}(x-1)$

or

$y=\frac{1}{3}x-\frac{4}{3}$

$y = \frac{1}{3}x - \frac{4}{3}$

For each problem, find the average rate of change of the function over the given interval.

5) $y = x^2 + 2;\ [-2, -\frac{3}{2}]$

$\left(\frac{-3}{2}\right)^2 + 2 =$

$-\frac{7}{2}$

$\frac{f(-\frac{3}{2}) - f(-2)}{\frac{-3}{2} - -2} = \frac{4.25 - 6}{.5} = \boxed{-3.5}$

6) $y = 2x^2 - 2x + 1;\ [-1, -\frac{1}{2}]$

-5

$= \frac{f(-\frac{1}{2}) - f(-1)}{-\frac{1}{2} + 1} = \frac{2.5 - 5}{+.5} = \boxed{-5}$

Skip

7) $y = -\frac{1}{x+2};\ [-1, -\frac{1}{2}]$

$\frac{2}{3}$

Skip

8) $y = 2x^2 + x + 2;\ [0, \frac{1}{2}]$

2

$\frac{f(\frac{1}{2}) - f(0)}{\frac{1}{2} - 0} = \frac{3 - 2}{.5} = \frac{1}{.5} = 2$

For each problem, find the equation of the secant line that intersects the given points on the function.

9) $y = -x^2 - 2;\ (1, -3), \left(\frac{3}{2}, -\frac{17}{4}\right)$

$m = \frac{\frac{-17}{4} + 3}{\frac{3}{2} - 1} = \frac{-1.25}{.5} = -2.5$

$y = -\frac{5}{2}x - \frac{1}{2}$ or $y + 3 = -2.5(x+1)$

10) $y = \frac{1}{x+3};\ \left(-1, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{2}{5}\right)$

$m = \frac{\frac{2}{5} - \frac{1}{2}}{-\frac{1}{2} + 1} = \frac{-.1}{.5} = -.2$

$y = -\frac{1}{5}x + \frac{3}{10}$ OR $y - \frac{1}{2} = -.2(x+1)$

Skip

11) $y = \frac{1}{x-1};\ \left(-2, -\frac{1}{3}\right), \left(-\frac{3}{2}, -\frac{2}{5}\right)$

$y = -\frac{2}{15}x - \frac{3}{5}$

Skip

12) $y = -\frac{1}{x};\ (1, -1), \left(\frac{3}{2}, -\frac{2}{3}\right)$

$y = \frac{2}{3}x - \frac{5}{3}$

Critical thinking question:

13) The police have accused a driver of breaking the speed limit of 60 miles per hour. As proof, they provide two photographs. One photo shows the driver's car passing a toll booth at exactly 6 PM. The second photo shows the driver's car passing another toll both 31 miles down the highway at exactly 6:30 PM. Does the photo evidence prove that the driver broke the speed limit during this time? $(6, 0)$ $(6.5, 31)$

Yes. The average rate of change is 62 mph, so the driver must have been breaking the speed limit some of the time.

$\frac{31 - 0}{6.5 - 6} = \frac{31}{.5} = 62$mph

Kuta Software - Infinite Calculus

Name Key

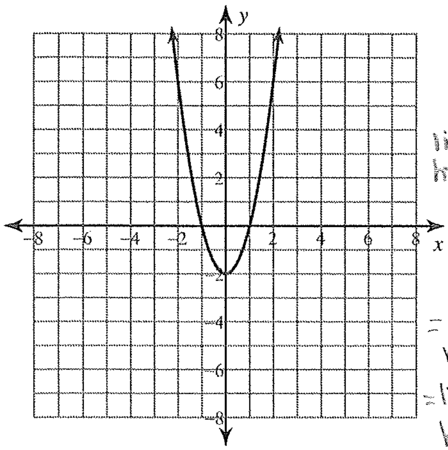
Instantaneous Rates of Change

Date________________ Period____

For each problem, find the average rate of change of the function over the given interval and also find the instantaneous rate of change at the leftmost value of the given interval.

1) $y = 2x^2 - 2$; $[1, \frac{3}{2}]$

Avg $\frac{f(\frac{3}{2}) - f(1)}{\frac{3}{2} - 1} = \frac{2.5 - 0}{.5} = 5$



instantaneous $x=1$

$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \frac{2(1+h)^2 - 2 - (0)}{h}$

$= \lim_{h \to 0} \frac{2h^2 + 4h + \cancel{2} - \cancel{2}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2h+4)}{\cancel{h}}$

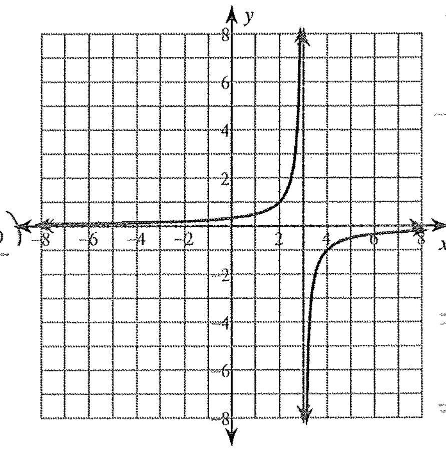
$= \lim_{h \to 0} 2h + 4 = 4$

$\boxed{\text{Average: } 5 \quad \text{Instant.: } 4}$

2) $y = -\frac{1}{x-3}$; $[0, \frac{1}{2}]$

Avg $\frac{f(\frac{1}{2}) - f(0)}{\frac{1}{2} - 0}$

$= \frac{.4 - \frac{1}{3}}{.5} = .1\overline{3}$ or $\frac{2}{15}$



instaneous at $x=0$

$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{\frac{-1}{h-3} - \frac{1}{3}}{h}$

$= \lim_{h \to 0} \frac{\frac{-3}{(h-3)3} - \frac{h-3}{3(h-3)}}{h}$

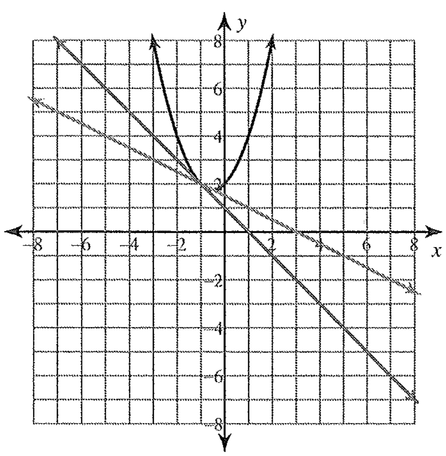
$= \lim_{h \to 0} \frac{\cancel{-3} - h + \cancel{3}}{3(h-3)} \cdot \frac{1}{h}$

$= \lim_{h \to 0} \frac{-1}{3(h-3)} = \frac{-1}{3(-3)} = \frac{1}{9}$

$\boxed{\text{Average: } \frac{2}{15} \quad \text{Instant.: } \frac{1}{9}}$

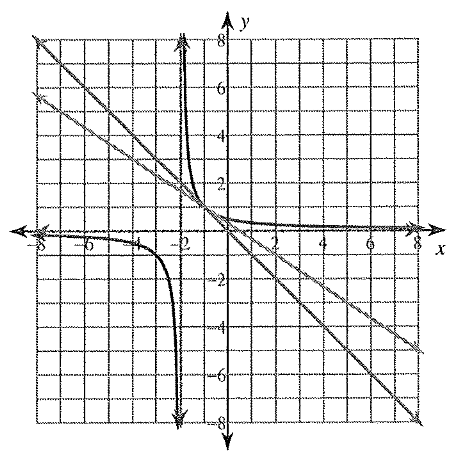
For each problem, find the equation of the secant line that intersects the given points on the function and also find the equation of the tangent line to the function at the leftmost given point. Sketch both lines for comparison.

3) $y = x^2 + x + 2$; $(-1, 2), \left(-\frac{1}{2}, \frac{7}{4}\right)$



Secant: $y = -\frac{1}{2}x + \frac{3}{2}$ Tangent: $y = -x + 1$

4) $y = \frac{1}{x+2}$; $(-1, 1), \left(-\frac{1}{2}, \frac{2}{3}\right)$



Secant: $y = -\frac{2}{3}x + \frac{1}{3}$ Tangent: $y = -x$