TEST

A Chapter 1

DATE Key

NAME

Directions: Show all steps leading to your answers, including any intermediate results obtained using a graphing utility. Use the back of the test or another sheet of paper if necessary.

1. Let L represent the line $y = \frac{1}{4}x + 3$. Write an equation for the line through (4, 0) that is **(a)** parallel to L, **(b)** perpendicular to L.

a) $m = \frac{1}{4}$ $0 = \frac{1}{4} \cdot 4 + b$ $0 = 1 + b$ $-1 = b$

b) $m = -4$ $0 = -4(4) + b$ $16 = b$

1. (a) $y = \frac{1}{4}x - 1$

(b) $y = -4x + 16$

2. Find the slope and y-intercept for the linear function described by the table.

x	−2	6	14
$f(x)$	−3	2	7

(6, 2) (14, 7)

$m = \frac{7-2}{14-6} = \frac{5}{8}$

$2 = \frac{5}{8} \cdot 6 + b$ $\quad 2 = \frac{15}{4} + b$

$\frac{8}{4} - \frac{15}{4} = b = \frac{-7}{4}$

2. Slope: $\frac{5}{8}$

y-intercept: $-\frac{7}{4}$

3. For the function $y = 5 - \sqrt{9 - x^2}$, **(a)** find the domain, **(b)** find the range, and **(c)** determine whether the function is odd, even or neither.

a) $9 - x^2 \geq 0$ $\quad 9 \geq x^2$ $\quad x^2 \leq 9$ $\quad -3 \leq x \leq 3$

3. (a) $[-3, 3]$

(b) $[2, 5]$

(c) even

4. Let $f(x) = \begin{cases} -0.5x, & x < -2 \\ \sqrt{x+2}, & x \geq -2 \end{cases}$

(a) Draw the graph of $f(x)$.

(b) Find the domain.

(c) Find the range.

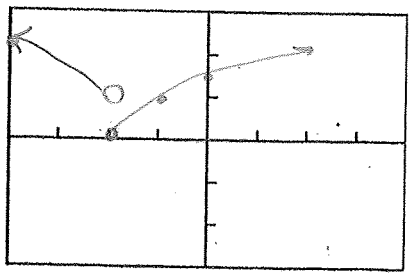
$x = -2$ $\quad -\frac{1}{2} \cdot \frac{-2}{1} = 1$

$x = -4$ $\quad -\frac{1}{2} \cdot \frac{-4}{1} = 2$

$y = -1$ $\quad \sqrt{-1+2} = 1$

$x = 0$ $\quad y = \sqrt{2}$

4. (a)



[−4, 4] by [−3, 3]

(b) $(-\infty, \infty)$

(c) $[0, \infty)$

5. Let $f(x) = x^2 + 5$ and $g(x) = \frac{1}{x}$.
Find formulas for **(a)** $f \circ g$ and **(b)** $f \circ g$.

$f \circ g = \left(\frac{1}{x}\right)^2 + 5$

$g \circ f = \frac{1}{x^2+5}$

5. (a) $f \circ g = \frac{1}{x^2} + 5$

(b) $g \circ f = \frac{1}{x^2+5}$

6. State the domain, range, and intercepts of the function $y = 9 - 3^{-x}$.

$9 = 9 - 3^{-x}$
$0 = -3^{-x}$
$0 = 3^{-x}$
$\log 0 = \log 3^{-x}$
und

x-int: $y = 0$
$0 = 9 - 3^{-x}$
$-9 = -3^{-x}$
$9 = 3^{-x}$
$\log 9 = -x \log 3$
$x = \frac{-\log 9}{\log 3} = -2$

y-int: $x = 0$
$y = 9 - 3^0$
$y = 8$

6. Domain: $(-\infty, \infty)$
Range: $(-\infty, 9)$
x-intercept(s): $(-\log 9/\log 3, 0)$ -2
y-intercept(s): $(0, 8)$

7. Solve the equation $4 - 3^x = 0$ graphically.

Graph. Find x-intercept.

7. ≈ 1.26185

8. In any given year, the population of a certain endangered species is reduced by 15%. If the population is now 7500, in how many years will the population be 4000?

85% remains
$y = 7500(.85)^x$
$4000 = 7500(.85)^x$

$.53 = .85^x$
$\log .53 = x \log .85$
$x = \frac{\log .53}{\log .85} = 3.8679$

8. 3.8679 yrs

9. Find a parametrization for the left half of the parabola $y = x^2 - 4x + 3$.

9. $x = t$ $\quad t \le 2$
$y = t^2 - 4t + 3$

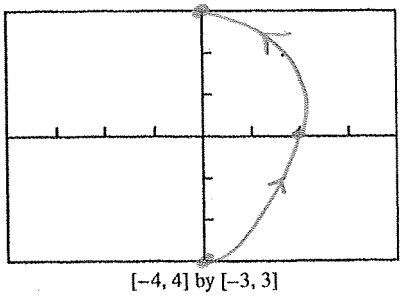
10. (a) Graph the parametrized curve described by $x = 2 \sin t$, $y = -3 \cos t$, $0 \le t \le \pi$. Indicate the direction in which the curve is traced.

$\frac{x}{2} = \sin t \qquad \frac{y}{-3} = \cos t$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

1) Square equations
2) Add equations
3) Key: $\sin^2 t + \cos^2 t = 1$

10. (a)



[−4, 4] by [−3, 3]

(b) Write a Cartesian equation for the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

(b) • $\frac{x^2}{4} + \frac{y^2}{9} = 1$
• right half

11. Let $f(x) = \sqrt[3]{x+2}$ and $g(x) = x^3 - 2$.
Which of the following are true?

I. $g(x) = f^{-1}(x)$ for all real values of x.

II. $(f \circ g)(x) = 1$ for all real values of x.

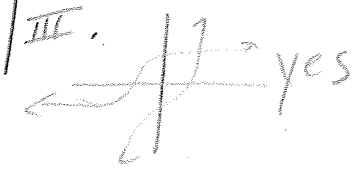
III. The function f is one-to-one.

(A) III only (B) I and II (C) I and III
(D) II and III (E) I, II, and III

11. C

I. $x = \sqrt[3]{y+2}$
$x^3 = y+2$
$x^3 - 2 = y$
yes

II. $f \circ g = \sqrt[3]{x^3 - 2 + 2}$
$= \sqrt[3]{x^3} = x$
False

III. 
yes

12. Let $f(x) = \dfrac{1}{x^2 - 4}$ for $x > 2$.
Find an expression for $f^{-1}(x)$.
(Be sure to state any necessary domain restrictions.)

$y > 0$

12. $y = \sqrt{\frac{1}{x} + 4}$; $x > 0$

$x = \frac{1}{y^2 - 4}$

$x(y^2 - 4) = 1$
$y^2 - 4 = \frac{1}{x}$
$y^2 = \frac{1}{x} + 4$

$y = \pm\sqrt{\frac{1}{x} + 4}$ $x > 0$
↑ Not (−)

13. The table gives Taiwan's nuclear power generation data in billions of kilowatt-hours. Let $x = 5$ represent 1980, $x = 10$ represent 1985, and so on.

Year	1980	1985	1990	1995
Energy produced	7.8	27.8	31.6	33.9

(a) Find a natural logarithm regression equation for the data.

(b) Predict when Taiwan's nuclear power generation will reach 40 billion kilowatt-hours. $x = 23.97$

13. **(a)** $y = -20.524 + 19.051 \cdot \ln x$
(b) ≈ 1999

$a = -20.5237\ldots$
$b = 19.051434\ldots$

14. An angle measuring $3\pi/8$ radians has its vertex at the center of a circle whose radius is 4 feet. Find the length of the subtended arc.

$\theta = \frac{s}{r}$

$\frac{3\pi}{8} = \frac{s}{4}$

$s = \frac{12\pi}{8} = \frac{3\pi}{2}$

14. $\frac{3\pi}{2} \approx 4.71$

15. Let $y = 5\cos(4(x-3)) - 1$. Determine the period, domain, and range of the function.

$P = \frac{2\pi}{4} = \frac{\pi}{2}$

15. Period: $\frac{\pi}{2}$
Domain: $(-\infty, \infty)$
Range: $[-6, 4]$

16. Solve the equation $\cot x = 4$ in the interval $0 \le x \le 2\pi$.

16. .245, 3.387