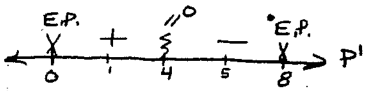
① $P = xy$
$P = (x)(8-x)$
$P = 8x - x^2$

$P' = 8 - 2x$
$8 - 2x = 0$
$2x = 8$
$x = 4$



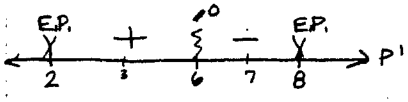
$P = xy$
$P = (4)(4)$
$P = 16$

$x + y = 8$
$y = 8 - x$

$y = 8 - 4$
$y = 4$

② $P = x^3y^2$
$P = x^3(10-x)^2$
$P = x^3(100 - 20x + x^2)$
$P = 100x^3 - 20x^4 + x^5$

$P' = 300x^2 - 80x^3 + 5x^4$
$300x^2 - 80x^3 + 5x^4 = 0$
$5x^2(60 - 16x + x^2) = 0$
$5x^2(x^2 - 16x + 60) = 0$
$5x^2(x-10)(x-6) = 0$
$\cancel{x=0}$ $\cancel{x=10}$ $x = 6$
not in domain



$P = x^3y^2$
$P = (6)^3(4)^2$
$P = (216)(16)$
$P = 3456$ ← MAXIMUM product

Since there were no critical points that formed a minimum, the minimum value mus
be at an endpoint.

$x = 2$: $P = x^3y^2$
$P = (2)^3(8)^2$
$P = (8)(64)$
$P = 512$ ← minimum product

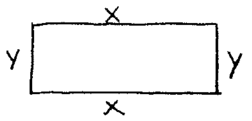
$x = 8$: $P = x^3y^2$
$P = (8)^3(2)^2$
$P = (512)(4)$
$P = 2048$

$x + y = 10$
$y = 10 - x$

$x \geq 2$, $y \geq 2$

$y = 10 - 6$
$y = 4$

③



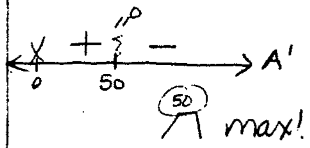
$P = 2x + 2y$
$200 = 2x + 2y$
$y = \frac{200-2x}{2}$

$y = 100 - x$

$A = xy$
$A = x(100-x)$
$A = 100x - x^2$

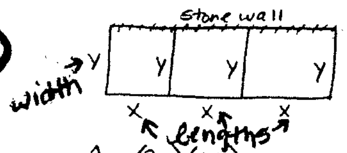
$A' = 100 - 2x$
$100 - 2x = 0$
$2x = 100$
$x = 50$ m.

$y = 100 - 50$
$y = 50$ m.



Maximum Area $= xy$
$= (50)(50)$
$= 2500\ m^2$

④



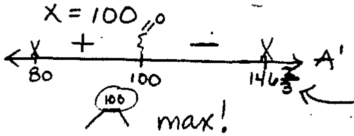
$P = 3x + 4y$
$600 = 3x + 4y$
$y = \frac{600-3x}{4}$

$x \geq 80$
$y \geq 40$

$A = (3x)(y)$
$A = (3x)\left(\frac{600-3x}{4}\right)$
$A = \frac{1800x - 9x^2}{4}$

$y = \frac{600-3(100)}{4}$
$y = \frac{300}{4}$
$y = 75$

$A' = \frac{1}{4}(1800 - 18x)$
$\frac{1}{4}(1800-18x) = 0$
$1800 - 18x = 0$
$18x = 1800$
$x = 100$



$40 = \frac{600-3x}{4}$
$160 = 600 - 3x$
$3x = 440$
$x = 146\frac{2}{3}$

*Maximum area $= 3xy$
$= 3(100)(75)$
$= 22500\ m^2$

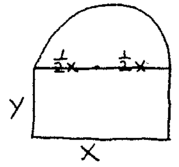
*Minimum area → must be at and endpoint

$x = 80$: $A = 3xy$
$= 3(80)(90)$
$= 21600\ m^2$

$x = 146\frac{2}{3}$: $A = 3xy$
$= 3(146\frac{2}{3})(40)$
$= 17600\ m^2$

The minimum area is $17600\ m^2$

⑤



$$P = \underbrace{x+2y}_{\text{rect.}} + \underbrace{\tfrac{1}{2}\pi d}_{\text{semi-cir.}}$$

$$P = x+2y+\tfrac{1}{2}\pi x$$

$$30 = x+2y+\tfrac{1}{2}\pi x$$

$$y = \frac{30-x-\frac{1}{2}\pi x}{2}$$

$$A = xy + \tfrac{1}{2}\pi r^2$$

$$A = xy + \tfrac{1}{2}\pi\left(\tfrac{1}{2}x\right)^2$$

$$A = xy + \tfrac{1}{8}\pi x^2$$

$$A = (x)\left(\frac{30-x-\frac{1}{2}\pi x}{2}\right) + \tfrac{1}{8}\pi x^2$$

$$A = \tfrac{1}{2}\left(30x - x^2 - \tfrac{1}{2}\pi x^2\right) + \tfrac{1}{8}\pi x^2$$

$$A' = \tfrac{1}{2}(30-2x-\pi x) + \tfrac{1}{4}\pi x$$

$$\tfrac{1}{2}(30-2x-\pi x) + \tfrac{1}{4}\pi x = 0$$

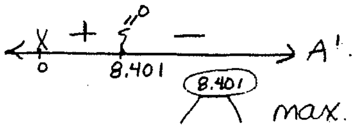
$$30-2x-\pi x+\tfrac{1}{2}\pi x = 0$$

$$30-2x-\tfrac{1}{2}\pi x = 0$$

$$30 = 2x+\tfrac{1}{2}\pi x$$

$$30 = x\left(2+\tfrac{1}{2}\pi\right)$$

$$x = \frac{30}{2+\frac{1}{2}\pi} = 8.401 \text{ ft.}$$



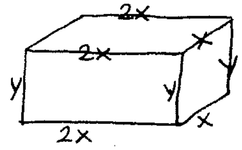
(Graph A' to get #line sig

$$y = \frac{30-8.401-\frac{1}{2}\pi(8.401)}{2}$$

$$y = 4.201 \text{ ft.}$$

Maximum area $= xy + \frac{1}{8}\pi x^2$
$= (8.401)(4.201) + \frac{1}{8}\pi(8.401)^2$
$= 63.008 \text{ ft}^2$

⑥



$$V = lwh$$

$$V = (2x)(x)(y)$$

$$V = 2x^2y$$

$$72 = 2x^2y$$

$$y = \frac{36}{x^2}$$

$$SA = \underset{\text{front/back}}{2[2xy]} + \underset{\text{top/bottom}}{2[2x^2]} + \underset{\text{left/right}}{2[xy]}$$

$$SA = 4xy + 4x^2 + 2xy$$

$$SA = 6xy + 4x^2$$

$$SA = 6x\left(\frac{36}{x^2}\right) + 4x^2$$

$$SA = \frac{216}{x} + 4x^2$$

$$SA' = \frac{-216}{x^2} + 8x$$

$$-\frac{216}{x^2} + 8x = 0$$

$$8x = \frac{216}{x^2}$$

$$8x^3 = 216$$

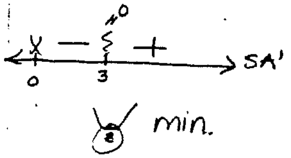
$$x^3 = 27$$

$$x = 3$$

$$y = \frac{36}{x^2}$$

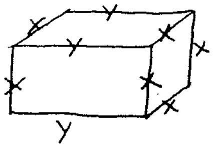
$$y = \frac{36}{3^2}$$

$$y = 4$$



Dimensions: 6ft. x 3ft. x 4ft.

⑦

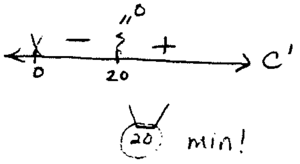


$V = lwh$
$V = (y)(x)(x)$
$V = x^2 y$
$6400 = x^2 y$
$y = \frac{6400}{x^2}$

$C = 3\underset{\text{base}}{[xy]} + 1\underset{\text{left/rt.}}{[2x^2]} + 1\underset{\text{front/back.}}{[2xy]}$
$C = 3xy + 2x^2 + 2xy$
$C = 5xy + 2x^2$
$C = 5x\left(\frac{6400}{x^2}\right) + 2x^2$
$C = \frac{32000}{x} + 2x^2$

$y = \frac{6400}{x^2}$
$y = \frac{6400}{20^2}$
$y = \frac{6400}{400}$
$y = 16$ ft.

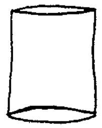
$C' = \frac{-32000}{x^2} + 4x$
$\frac{-32000}{x^2} + 4x = 0$
$4x = \frac{32000}{x^2}$
$4x^3 = 32000$
$x^3 = 8000$
$x = 20$ ft.



min!

Dimensions: 16 ft × 20 ft × 20 ft.

⑧

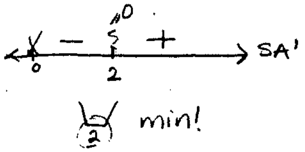


$V = \pi r^2 h$
$16\pi = \pi r^2 h$
$16 = r^2 h$
$h = \frac{16}{r^2}$

$SA = 2\pi r^2 + 2\pi r h$
$SA = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$
$SA = 2\pi r^2 + \frac{32\pi}{r}$

$h = \frac{16}{r^2}$
$h = \frac{16}{2^2}$
$h = \frac{16}{4}$
$h = 4$ in.

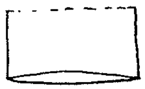
$SA' = 4\pi r - \frac{32\pi}{r^2}$
$4\pi r - \frac{32\pi}{r^2} = 0$
$4\pi r = \frac{32\pi}{r^2}$
$4\pi r^3 = 32\pi$
$4r^3 = 32$
$r^3 = 8$
$r = 2$ in.



min!

Dimensions: radius = 2 in.
height = 4 in.

⑨



$V = \pi r^2 h$

$500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$

$h = \frac{500}{\pi r^2}$

$h = \frac{500}{\pi (5.419)^2}$

$h = 5.420$ ft.

$T = 15(\pi r^2 + 2\pi r h)$

$T = 15(\pi r^2 + 2\pi r (\frac{500}{\pi r^2}))$

$T = 15(\pi r^2 + \frac{1000}{r})$

$T' = 15(2\pi r - \frac{1000}{r^2})$

$15(2\pi r - \frac{1000}{r^2}) = 0$

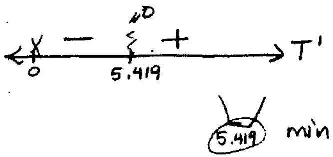
$2\pi r - \frac{1000}{r^2} = 0$

$2\pi r = \frac{1000}{r^2}$

$2\pi r^3 = 1000$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}} = 5.419$ ft.



min!

Dimensions (Size): radius = 5.419 ft.
height = 5.420 ft.

$\text{Time} = 15(\pi r^2 + \frac{1000}{r})$

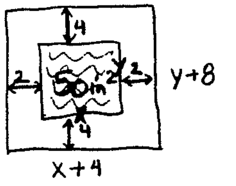
$= 15(\pi (5.419)^2 + \frac{1000}{5.419})$

= 4151.858 minutes

= 69.198 hours

= 2.883 days.

⑩



$50 = xy$

$y = \frac{50}{x}$

$y = \frac{50}{5}$

$y = 10$

$A = (x+4)(y+8)$

$A = (x+4)(\frac{50}{x} + 8)$

$A = 50 + 8x + \frac{200}{x} + 32$

$A = \frac{200}{x} + 8x + 82$

$A' = \frac{-200}{x^2} + 8$

$0 = \frac{-200}{x^2} + 8$

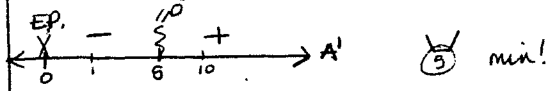
$\frac{200}{x^2} = 8$

$8x^2 = 200$

$x^2 = 25$

$x = \cancel{\pm} 5$

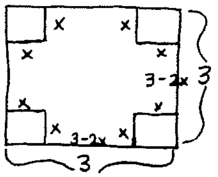
↑ no negative lengths.



min!

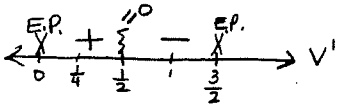
Dimensions: 9 in. × 18 in.

⑪



$V = lwh$
$V = (3-2x)(3-2x)(x)$
$V = (9-12x+4x^2)(x)$
$V = 9x - 12x^2 + 4x^3$

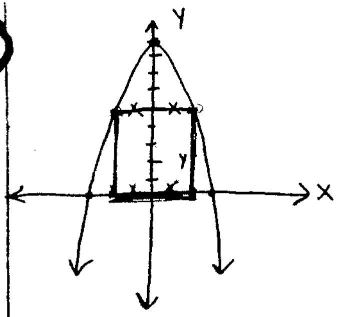
$V' = 9 - 24x + 12x^2$
$12x^2 - 24x + 9 = 0$
$3(4x^2 - 8x + 3) = 0$
$3(4x^2 - 2x - 6x + 3) = 0$
$3(2x(2x-1) - 3(2x-1)) = 0$
$3(2x-1)(2x-3) = 0$
$x = \frac{1}{2} \quad x = \frac{3}{2}$



max!

$V = (3-2x)(3-2x)(x)$
$V = (3-1)(3-1)(\frac{1}{2})$
$V = (2)(2)(\frac{1}{2})$
$\boxed{V = 2 ft^3}$

⑫

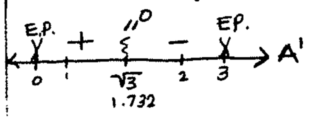


$y = -x^2 + 9$
$0 = -x^2 + 9$
$x^2 = 9$
$x = \pm 3$ ← zeros

$y = -(\sqrt{3})^2 + 9$
$y = -3 + 9$
$y = 6$

$A = bh$
$A = (2x)(y)$
$A = (2x)(-x^2+9)$
$A = -2x^3 + 18x$

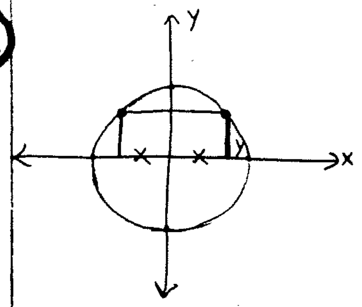
$A' = -6x^2 + 18$
$0 = -6x^2 + 18$
$6x^2 = 18$
$x^2 = 3$
$x = \cancel{\pm}\sqrt{3}$
↑ no negative lengths.



max!

$A = 2xy$
$A = 2(\sqrt{3})(6)$
$\boxed{A = 12\sqrt{3} = 20.785 \text{ units}^2}$

⑬



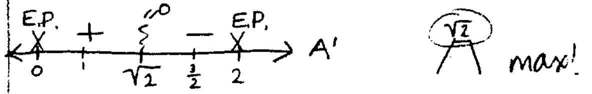
$x^2+y^2=4$
center $(0,0)$
radius = 2.

$x^2+y^2=4$
$y^2 = 4-x^2$
$y = \cancel{\pm}\sqrt{4-x^2}$

$A = bh$
$A = (2x)(y)$
$A = (2x)(\sqrt{4-x^2})$

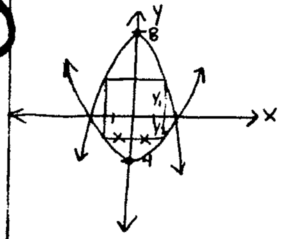
$A' = (2x)(\frac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x)) + (\sqrt{4-x^2})(2)$
$\frac{-2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2} = 0$
$-2x^2 + 2(4-x^2) = 0$
$-2x^2 + 8 - 2x^2 = 0$
$-4x^2 + 8 = 0$
$4x^2 = 8$
$x^2 = 2$
$x = \cancel{\pm}\sqrt{2}$
↑ no negative lengths.

$y = \sqrt{4-\sqrt{2}^2}$
$y = \sqrt{4-2}$
$y = \sqrt{2}$



$A = 2xy$
$A = 2(\sqrt{2})(\sqrt{2})$
$\boxed{A = 4 \text{ units}^2}$

⑭

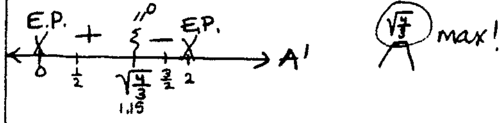


Y_1 : $y = 8-2x^2$
$0 = 8-2x^2$
$2x^2 = 8$
$x^2 = 4$
$x = \pm 2$ ← zeros

Y_2 : $y = x^2-4$
$0 = x^2-4$
$x^2 = 4$
$x = \pm 2$ ← zeros

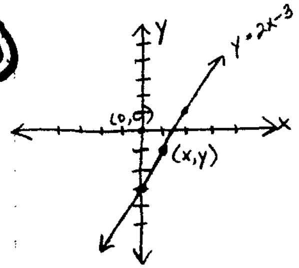
$A = bh$
$A = (2x)(y_1 - y_2)$ ← this will have a negative value. To make it positive (distance), subtract the negative value.
$A = (2x)(8-2x^2-(x^2-4))$
$A = (2x)(8-2x^2-x^2+4)$
$A = (2x)(12-3x^2)$
$A = 24x - 6x^3$

$A' = 24 - 18x^2$
$24 - 18x^2 = 0$
$6(4-3x^2) = 0$
$x = \cancel{\pm}\sqrt{\frac{4}{3}}$
↑ no negative lengths.



$A = (2x)(12-3x^2)$
$A = (2\sqrt{\frac{4}{3}})(12-3(\sqrt{\frac{4}{3}}^2)$
$A = (\frac{4}{\sqrt{3}})(12-4)$
$\boxed{A = \frac{32}{\sqrt{3}} = 18.475 \text{ units}^2}$

⑯



$y = 2x - 3$ ← helper

$y = 2\left(\frac{6}{5}\right) - 3$

$y = \frac{12}{5} - 3$

$y = \frac{12}{5} - \frac{15}{5}$

$y = \frac{-3}{5}$

$d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d = \sqrt{(y-0)^2+(x-0)^2}$

$d = \sqrt{y^2+x^2}$

$d^2 = y^2+x^2$

$d^2 = (2x-3)^2+x^2$

$d^2 = 4x^2-12x+9+x^2$

$d^2 = 5x^2-12x+9$

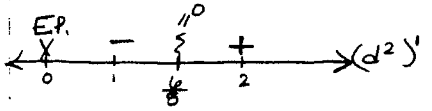
If distance is at a minimum, then the square of the distance is also a minimum. Use this idea to make the algebra simpler

$(d^2)' = 10x-12$

$10x-12=0$

$2(5x-6)=0$

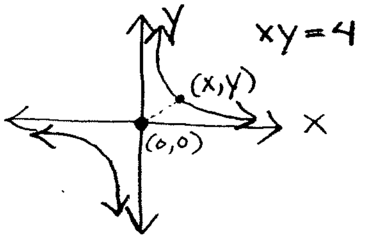
$x = \frac{6}{5}$



min!

The point on the line that is closest to the origin is $\left(\frac{6}{5}, \frac{-3}{5}\right)$.

⑰



$xy = 4$

$y = \frac{4}{x}$

$D = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$D = \sqrt{(y-0)^2+(x-0)^2}$

$D = \sqrt{y^2+x^2}$

$D = \sqrt{\left(\frac{4}{x}\right)^2+x^2}$

$D = \sqrt{\frac{16}{x^2}+x^2}$

$D^2 = \frac{16}{x^2}+x^2$

$(D^2)' = \frac{(x^2)(0)-(16)(2x)}{x^4} + 2x$

$(D^2)' = \frac{-32}{x^3} + 2x$

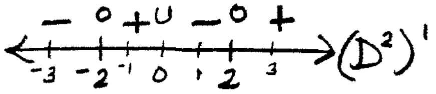
$\frac{-32}{x^3} + 2x = 0$

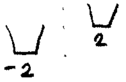
$2x = \frac{32}{x^3}$

$2x^4 = 32$

$x^4 = 16$

$x = \pm 2$





mins!

$y = \frac{4}{x}$

$y = \frac{4}{2}$

$y = 2$

$y = \frac{4}{x}$

$y = \frac{4}{-2}$

$y = -2$ ← Gives the same distance as $x = 2, y = 2$.

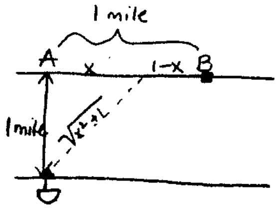
$D = \sqrt{y^2+x^2}$

$D = \sqrt{(2)^2+(2)^2}$

$D = \sqrt{4+4} = \sqrt{8}$

Minimum distance = $\sqrt{8}$

⑱



$T_{water} = \sqrt{x^2+1}$ miles × $\frac{1 \text{ hr}}{3 \text{ miles}}$ = $\frac{\sqrt{x^2+1}}{3}$ hours.

$T_{land} = (1-x)$ miles × $\frac{1 \text{ hr}}{5 \text{ miles}}$ = $\frac{1-x}{5}$ hours.

$T = \frac{\sqrt{x^2+1}}{3} + \frac{1-x}{5}$

$T' = \frac{1}{3}\left(\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right) + \frac{1}{5}(-1)$

$T' = \frac{x}{3\sqrt{x^2+1}} - \frac{1}{5}$

$\frac{x}{3\sqrt{x^2+1}} - \frac{1}{5} = 0$

$\frac{x}{3\sqrt{x^2+1}} = \frac{1}{5}$

$3\sqrt{x^2+1} = 5x$

$9(x^2+1) = 25x^2$

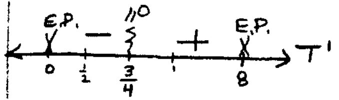
$9x^2+9 = 25x^2$

$9 = 16x^2$

$\frac{9}{16} = x^2$

$x = \overset{+}{\cancel{-}}\frac{3}{4}$

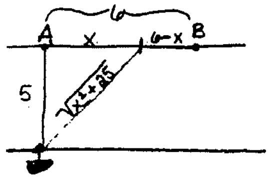
↑ no negative distances



$\frac{3}{4}$ min!

The boat should land $\frac{3}{4}$ miles from point A

⑲



$T_{water} = \sqrt{x^2+25}$ miles × $\frac{1 \text{ hour}}{2 \text{ miles}}$ = $\frac{\sqrt{x^2+25}}{2}$ hours.

$T_{land} = 6-x$ miles × $\frac{1 \text{ hour}}{4 \text{ miles}}$ = $\frac{6-x}{4}$ hours.

$T = \frac{\sqrt{x^2+25}}{2} + \frac{6-x}{4}$

$T' = \frac{1}{2}\left(\frac{1}{2}(x^2+25)^{-\frac{1}{2}}(2x)\right) + \frac{1}{4}(-1)$

$T' = \frac{x}{2\sqrt{x^2+25}} - \frac{1}{4}$

$\frac{x}{2\sqrt{x^2+25}} - \frac{1}{4} = 0$

$\frac{x}{2\sqrt{x^2+25}} = \frac{1}{4}$

$2\sqrt{x^2+25} = 4x$

$4(x^2+25) = 16x^2$

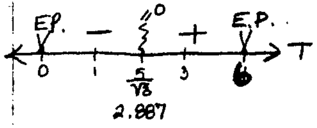
$4x^2+100 = 16x^2$

$100 = 12x^2$

$x^2 = \frac{25}{3}$

$x = \overset{+}{\cancel{-}}\frac{5}{\sqrt{3}}$

↑ no negative distances.



$\frac{5}{\sqrt{3}}$ min!

The man should land his boat 2.887 miles from point A.

⑳

$y = ax^3 + bx^2 + cx + d$

$y' = 3ax^2 + 2bx + c$

Local max. at (−1, 10) means...

$3a(-1)^2 + 2b(-1) + c = 0$

* $3a - 2b + c = 0$

$y'' = 6ax + 2b$

Inflection point at (1, −6) means...

$6a(1) + 2b = 0$

* $6a + 2b = 0$

(−1, 10) means...

$10 = a(-1)^3 + b(-1)^2 + c(-1) + d$

* $10 = -a + b - c + d$

(1, −6) means...

$-6 = a(1)^3 + b(1)^2 + c(1) + d$

* $-6 = a + b + c + d$

$$\begin{array}{rl} & 10 = -a + b - c + d \\ -1(& \underline{-6 = a + b + c + d}) \end{array}$$

← Try to eliminate d since it is not in the y' or y'' equations.

$$\begin{array}{rl} & 10 = -a + b - c + d \\ + & \underline{6 = -a - b - c - d} \\ & 16 = -2a - 2c \end{array}$$

*over→

$16 = -2a - 2c$

$2c = -2a - 16$

$c = -a - 8$

$3a - 2b + c = 0$

$3a - 2b + (-a - 8) = 0$

$2a - 2b = 8$

$$\begin{array}{rl} & 2a - 2b = 8 \\ + & \underline{6a + 2b = 0} \\ & 8a = 8 \\ & \boxed{a = 1} \end{array}$$

$c = -a - 8$

$c = -1 - 8$

$\boxed{c = -9}$

$2a - 2b = 8$

$2(1) - 2b = 8$

$-2b = 6$

$\boxed{b = -3}$

$10 = -a + b - c + d$

$10 = -1 - 3 + 9 + d$

$10 = 5 + d$

$\boxed{d = 5}$

㉑ $f(x) = 3x^2 - x^3$

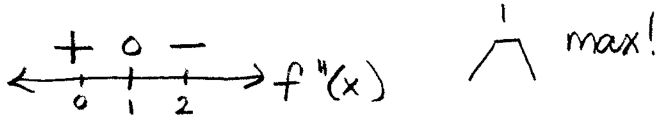
$f'(x) = 6x - 3x^2$ ← Maximize

$f''(x) = 6 - 6x$

$6 - 6x = 0$

$6x = 6$

$x = 1$



$f'(1) = 6(1) - 3(1)^2$

$= 6 - 3$

$= 3$

The maximum value
of the derivative
is f'(1) = 3.