

LIMITS AT NON-BREAKING POINTS

1) $\lim_{x \rightarrow 1} x^3 + 2x - 5 = \boxed{-2}$
 $1^3 + 2(1) - 5 =$

2) $\lim_{x \rightarrow 9} \frac{x^2 - 9}{x - 3} = \boxed{12}$
 $\frac{81 - 9}{9 - 3} = \frac{72}{6} = 12$

3) $\lim_{x \rightarrow 27} \sqrt[3]{x} = \boxed{3}$
 $\sqrt[3]{27}$

4) $\lim_{x \rightarrow 3} \frac{-1}{x} = \boxed{-\frac{1}{3}}$

5) $\lim_{x \rightarrow 2} \sqrt{x+2} = \boxed{2}$
 $\sqrt{2+2}$

6) $\lim_{x \rightarrow 1} \frac{x-1}{x+1} = \boxed{0}$
 $\frac{1-1}{1+1} = \frac{0}{2}$

HOLES IN THE GRAPH

7) $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2} = \boxed{7}$
 $\frac{(x+5)(x-2)}{x-2} = x+5$
 $= 2+5$

8) $\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x - 1} = \boxed{2}$
 $\frac{(x+1)(x-1)}{x-1} = x+1$
 $1+1$

9) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{4 - x} = \boxed{-8}$
 $\frac{(x+4)(x-4)}{-1(x-4)} = \frac{x+4}{-1} = \frac{8}{-1} = -8$

L'Hop 10) $\lim_{x \rightarrow 2} \frac{\sqrt{x+11} - 3}{x+2} = \boxed{\frac{1}{6}}$
 $\lim_{x \rightarrow 2} \frac{\sqrt{x+11} - 3}{x+2} = \lim_{x \rightarrow 2} \frac{\frac{1}{2}(x+11)^{-\frac{1}{2}}}{1} =$
 $= \frac{1}{2\sqrt{-2+11}} = \frac{1}{6}$

11) $\lim_{x \rightarrow 64} \frac{8 - \sqrt{x}}{x - 64} = \boxed{-\frac{1}{16}}$
 $\lim_{x \rightarrow 64} \frac{-\frac{1}{2}x^{-\frac{1}{2}}}{1} = \frac{-1}{2\sqrt{64}} = \frac{-1}{16}$

12) $\lim_{x \rightarrow 7} \frac{x^2 - 49}{\sqrt{x+18} - 5} = \boxed{140}$
 $\lim_{x \rightarrow 7} \frac{2x}{\frac{1}{2}(x+18)^{-\frac{1}{2}}} = \frac{2 \cdot 7 \sqrt{7+18} \cdot 2}{1} = 140$

RADICALS

13) $\lim_{x \rightarrow 3^+} \sqrt{3-x} = \boxed{0}$
 plug in 2.99

14) $\lim_{x \rightarrow 5^-} \sqrt{x+5} = \boxed{DNE}$

15) $\lim_{x \rightarrow 4^+} \sqrt{x^2 - 16} + 1 = \boxed{1}$
 $0 + 1$

16) $\lim_{x \rightarrow 5^+} \sqrt{25-x^2} = \boxed{DNE}$
 5.1 ↗

17) $\lim_{x \rightarrow 5^-} \sqrt{25-x^2} = 0$
 4.9 ↗

18) $\lim_{x \rightarrow 5} \sqrt{25-x^2} = 0$
 $\sqrt{25-5^2} = 0$

ASYMPTOTES

19) $\lim_{x \rightarrow 2^+} \frac{3}{x-2} = \boxed{\infty}$
 No limit
 2.1 ↗

20) $\lim_{x \rightarrow 2^-} \frac{-5}{2-x} = \boxed{-\infty}$
 No limit
 1.99 ↗

21) $\lim_{x \rightarrow -8} \frac{x}{x+8} = \boxed{DNE}$
 $-8.1 \rightarrow \frac{-}{+} = +$
 $-7.9 \rightarrow \frac{-}{-} = -$

22) $\lim_{x \rightarrow -3^+} \frac{3x}{x+3} = \boxed{-\infty}$
 No limit
 -2.9 ↗

23) $\lim_{x \rightarrow 6} \frac{-8}{(x-6)^2} = \boxed{-\infty}$
 No limit
 6.1 ↗ $\frac{-}{+} = -$
 5.9 ↗ $\frac{-}{+} = -$

24) $\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = \boxed{-\infty}$
 No limit

TRIG. FUNCTIONS

25) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \boxed{1}$ 26) $\lim_{x \rightarrow 0} \frac{\cos x}{x} = \boxed{\text{DNE}}$ 27) $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \boxed{1}$ 28) $\lim_{x \rightarrow 0} \frac{\sin x \tan x}{x^2} = \boxed{1}$

$\frac{\sin x}{x} \cdot \frac{\tan x}{x} = 1 \cdot 1$

29) $\lim_{x \rightarrow 0} \frac{3 \sin 3x}{8x} = \boxed{\frac{9}{8}}$ 30) $\lim_{x \rightarrow 0} \frac{6 \sin x \cos x}{5x} = \boxed{\frac{6}{5}}$ 31) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{5 \sin 3x}{x} = \boxed{\frac{-10}{\pi}}$

$\frac{3 \cdot \cos 3x \cdot 3}{8} = \frac{1 \cdot 3 \cdot 3}{8}$

$\frac{6}{5} \frac{\sin x}{x} \cdot \cos x = \frac{6}{5} \cdot 1 \cdot 1$

$\frac{5 \sin \frac{3\pi}{2}}{\frac{\pi}{2}} = \frac{-5 \cdot 2}{\pi}$

PIECE-WISE FUNCTIONS

$$f(x) = \begin{cases} 3-x & x < -3 \\ 2x+1 & -3 \leq x < 4 \\ 9 & x \geq 4 \end{cases}$$

32) $\lim_{x \rightarrow -5} f(x) = \boxed{8}$ 33) $\lim_{x \rightarrow -3^+} f(x) = \boxed{5}$ 34) $\lim_{x \rightarrow -3^-} f(x) = \boxed{6}$ 35) $\lim_{x \rightarrow -3} f(x) = \boxed{\text{ONE}}$

36) $\lim_{x \rightarrow 4^+} f(x) = \boxed{9}$ 37) $\lim_{x \rightarrow 4^-} f(x) = \boxed{9}$ 38) $\lim_{x \rightarrow 4} f(x) = \boxed{9}$ 39) $\lim_{x \rightarrow 7^+} f(x) = \boxed{9}$

LIMITS THAT APPROACH INFINITY

40) $\lim_{x \rightarrow \infty} \frac{3-7x^2}{14x^2+1} = \boxed{\frac{-1}{2}}$ 41) $\lim_{x \rightarrow \infty} \frac{x^3+5}{-x^2} = \boxed{-\infty}$ 42) $\lim_{x \rightarrow \infty} \frac{3-2x}{x^2+9} = \boxed{0}$ 43) $\lim_{x \rightarrow \infty} \frac{4x-5}{7x+1} = \boxed{\frac{4}{7}}$

44) $\lim_{x \rightarrow \infty} \frac{3-x}{x-3} = \boxed{-1}$ 45) $\lim_{x \rightarrow \infty} 5 = \boxed{5}$ 46) $\lim_{x \rightarrow \infty} \frac{2}{5-x} = \boxed{0}$ 47) $\lim_{x \rightarrow \infty} \frac{7x^5+3}{3x^2+5} = \boxed{-\infty}$

Find the vertical asymptotes and holes for each.

$(x+5)(x-3)$

48) $f(x) = \frac{3x-2}{5-x}$

49) $f(x) = \frac{x^2+2x-15}{x-3}$

50) $f(x) = \frac{8-x}{8-x}$

vert. asym. $\boxed{x=5}$ hole $\boxed{\text{None}}$

vert. asym. $\boxed{\text{None}}$ hole $\boxed{x=3}$

vert. asym. $\boxed{\text{None}}$ hole $\boxed{x=8}$

51) $f(x) = \frac{8x-1}{3x^2+5}$

52) $f(x) = \frac{(x+2)(x-3)(x+4)}{(x+2)(x+4)(x+7)}$

53) $f(x) = \frac{(2x+1)(x+3)}{(x-4)(2x+1)}$

vert. asym. $\boxed{\text{None}}$ hole $\boxed{\text{None}}$

vert. asym. $\boxed{x=-7}$ hole $\boxed{x=-2, x=-4}$

vert. asym. $\boxed{x=4}$ hole $\boxed{x=-\frac{1}{2}}$

Write an equation that meets the requirements.

54) Hole at: $x=3$

vert. asym.: $x=-5$

$$y = \frac{x-3}{(x-3)(x+5)}$$

55) Hole at: $(2, 5)$

$$y = \frac{(x-2)(x+3)}{(x-2)}$$

$5 = x + ?$

56) vert. asym.: $x=-4$

Hole at $(-2, \frac{-5}{2})$

$$\frac{x+3-5}{x+4} = \frac{x-2}{x+4}$$

$x=-2$

$$y = \frac{(x+2)(x-3)}{(x+4)(x+2)}$$

57) $\lim_{x \rightarrow 0} f(x) = \frac{1}{2}$ vert. asym.: $x = \frac{-7}{2}$

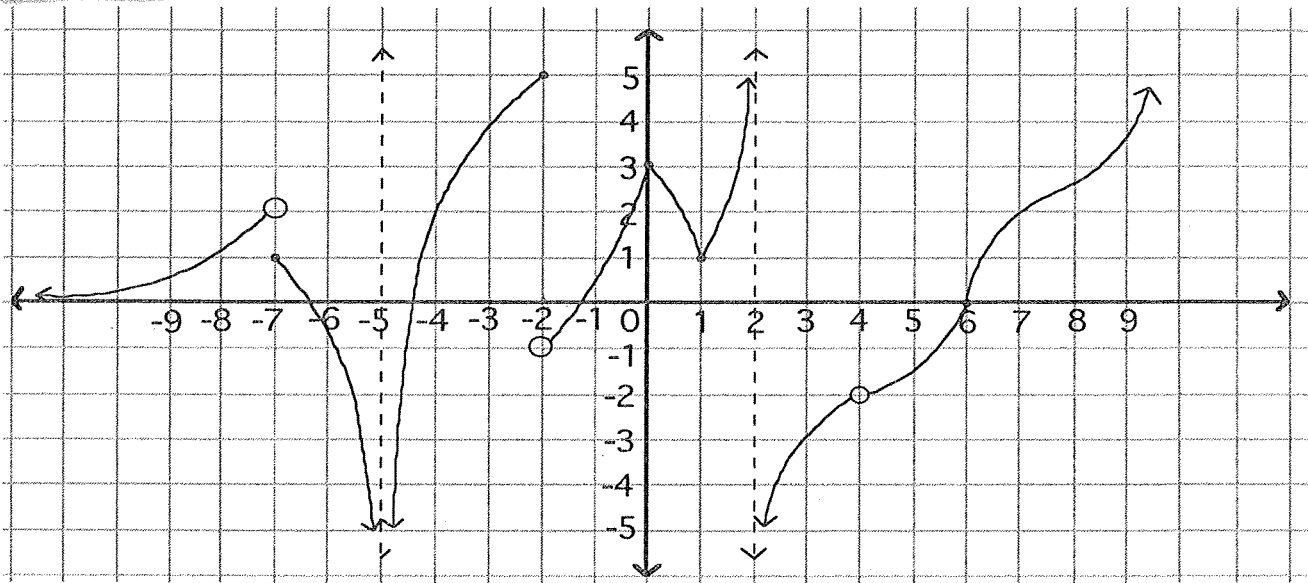
$x + \frac{7}{2}$

$$y = \frac{7}{2(2x+7)}$$

when $x=0$ $\frac{7}{2}$

58) $\lim_{x \rightarrow \infty} f(x) = 3$ vert. asym.: $x = -2$

$$y = \frac{3x}{x+2}$$



59) $\lim_{x \rightarrow -2^+} f(x) = -1$

60) $\lim_{x \rightarrow -2^-} f(x) = 5$

61) $\lim_{x \rightarrow 0} f(x) = 3$

62) $\lim_{x \rightarrow 2} f(x) = \text{DNE}$

63) $\lim_{x \rightarrow 4} f(x) = -2$

64) $\lim_{x \rightarrow 6} f(x) = 0$

65) $\lim_{x \rightarrow -7^-} f(x) = 2$

66) $\lim_{x \rightarrow \infty} f(x) = \infty$

67) $\lim_{x \rightarrow -\infty} f(x) = 0$

68) $\lim_{x \rightarrow -5} f(x) = -\infty$
No limit

69) $\lim_{x \rightarrow -7} f(x) = \text{DNE}$

70) $\lim_{x \rightarrow -2} f(x) = \text{DNE}$

71) $\lim_{x \rightarrow 2^+} f(x) = -\infty$

72) $\lim_{x \rightarrow 2^-} f(x) = \infty$

73) $\lim_{x \rightarrow 1} f(x) = 1$

74) $\lim_{x \rightarrow -1} f(x) = \frac{1}{2}$

75) $\lim_{x \rightarrow -7^+} f(x) = 1$

76) $\lim_{x \rightarrow -5^-} f(x) = -\infty$

77) $\lim_{x \rightarrow -5^+} f(x) = -\infty$

78) $\lim_{x \rightarrow 4^+} f(x) = -2$

79) The graph is discontinuous at which points: $x = -7, -5, -2, 2, 4$

80) $\lim_{x \rightarrow 1^+} \frac{\sqrt{x-1}}{x-1} = \infty$

81) $\lim_{x \rightarrow 2^+} \frac{(x-2)^2}{x-2} = 0$

$$\frac{\sqrt{x-1}}{x-1} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} = \frac{x-1}{x-1\sqrt{x-1}} = \frac{1}{\sqrt{x-1}}$$

$$\frac{(x-2)(x-2)}{x-2} = 0$$