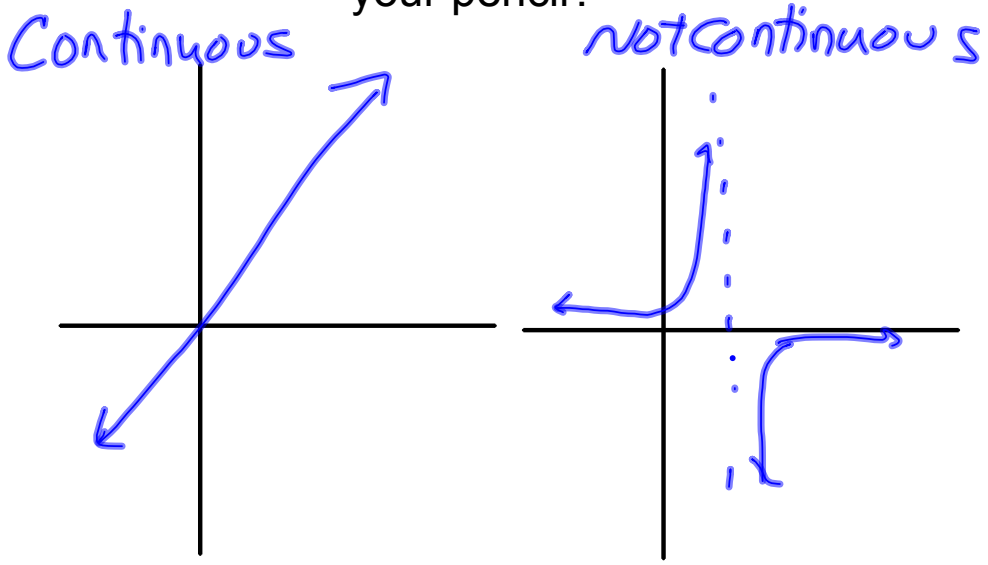
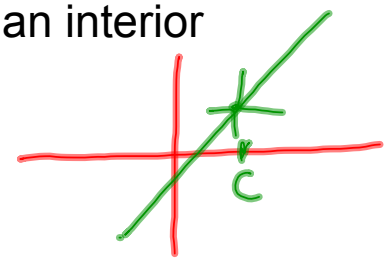
2.3 Continuity - (Continuous)

Can you trace the graph without lifting your pencil?



Interior Points

A function f(x) is continuous at an interior point if $\lim_{x \to c} f(x) = f(c)$



Endpoints

A function f(x) is continuous at the left endpoint if $\lim_{x \to a+} f(x) = f(a)$

A function f(x) is continuous at the right endpoint if $\lim_{x \to a-} f(x) = f(a)$

4 types of discontinuity

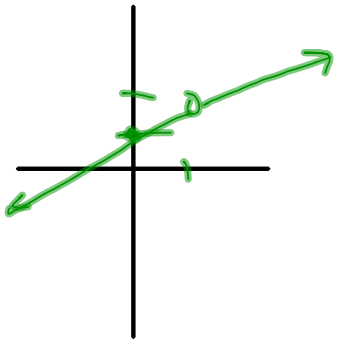
1. Removable (Hole in the graph)

- Can be removed by filling in the missing point

Ex 1) $f(x) = \frac{x^2 - 1}{x - 1}$

$= \frac{(x-1)(x+1)}{x-1}$

$x = 1$, hole

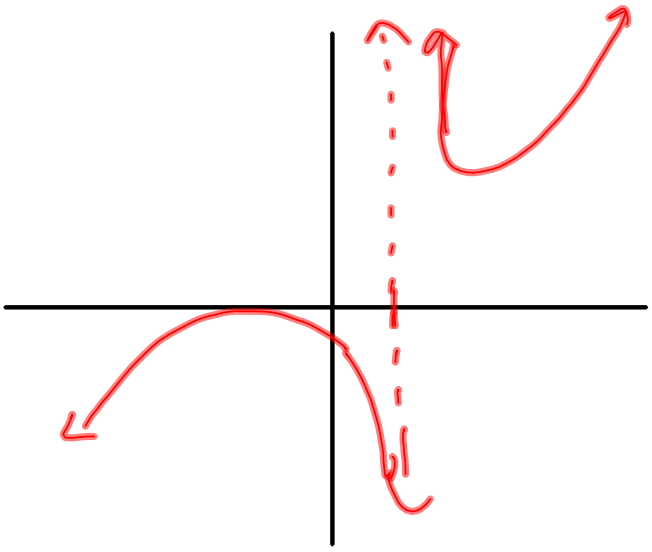


2. Infinite (Vertical Asymptote)

$(x+1)(x+1)$

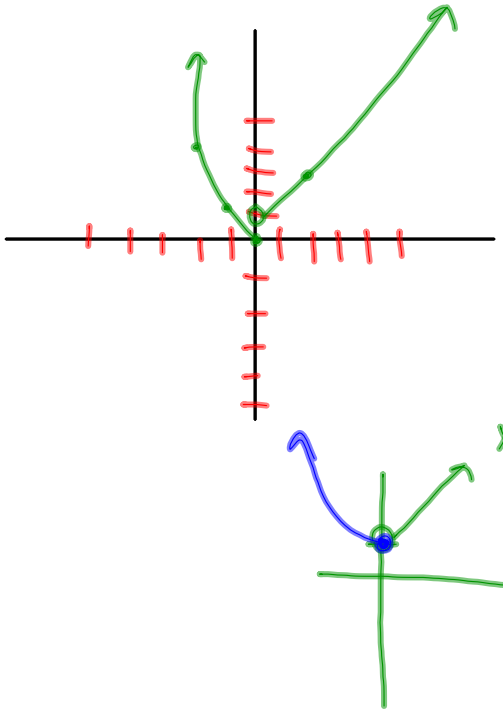
Ex 2) $f(x) = \frac{x^2 + 2x + 1}{x - 1}$

VA: $x = 1$



3. Jump (Piecewise Functions)

Ex 3) $f(x) = \begin{cases} x + 1, & x > 0 \\ x^2, & x \leq 0 \end{cases}$



Ex 3.5) Find "a" so that f(x) is continuous of:

$f(x) = \begin{cases} x + 1, & x > 0 \\ x^2 + a, & x \leq 0 \end{cases}$

$x=0 \quad 0+1=1$

$x^2+a=1$

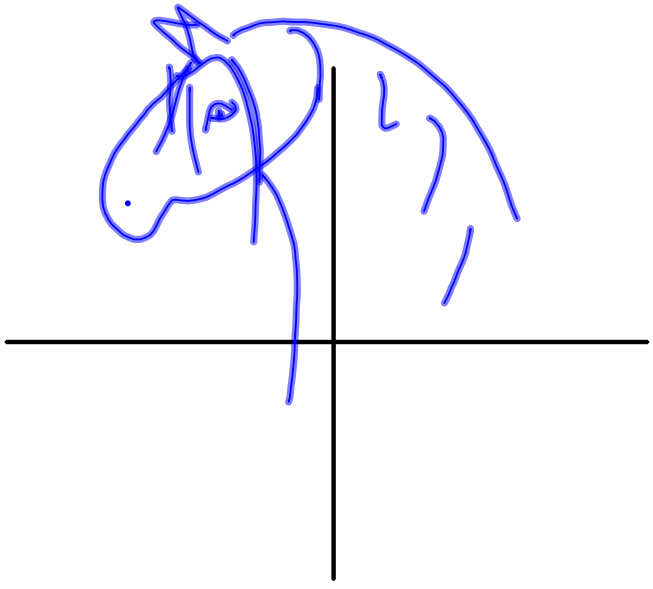
$0^2+a=1$

$a=1$

x^2+1

4. Oscillating

Ex 4) $f(x) = \sin(1/x)$



See page 80 "f" for graph

Intermediate Value Theorem

If a function is continuous on the interval [a,b], then f(x) must take on all y-values between f(a) and f(b)