2.3 WKST

AP Calculus AB
2.3 Continuity Worksheet

Name Key

State whether or not each of the following functions is continuous. If not, state where the discontinuity occurs and whether or not it is removable. Is the discontinuity is an asymptote, a hole, or a jump? If it is an asymptote, what is its equation?

1) $f(x) = \dfrac{x}{x^2+1}$

- continuous; no place where function is undefined

2) $f(x) = \dfrac{x}{2x^2-x-1}$

$(2x+1)(x-1)$

- Not continuous; Asymptotes at $x = -\frac{1}{2}$, $x = 1$
- Not removable

3) $f(x) = \dfrac{2x+3}{x^2-x-6}$

$(x-3)(x+2)$

Not continuous; Asymptotes at $x=3, x=-2$

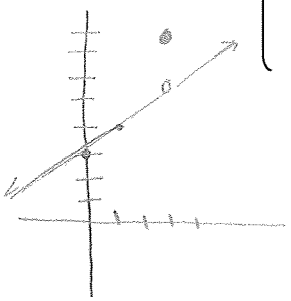
4) $f(x) = \dfrac{x-4}{x^2-16} = \dfrac{1}{x+4}$

$(x-4)(x+4)$

- Not continuous; hole at $x=4$, asymptote at $x=-4$
- removable if $f(4) = \frac{1}{8}$

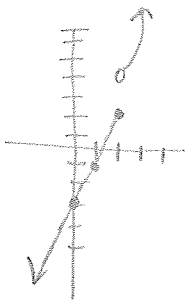
5) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 8 & \text{if } x = 3 \end{cases}$

$\dfrac{(x-3)(x+3)}{x-3} = 3+3 = 6$



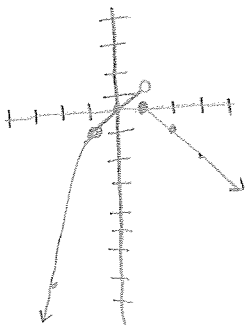
- Not continuous; hole at $x=3$
- removable

6) $f(x) = \begin{cases} 2x-3 & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$



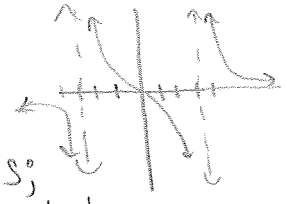
- Not continuous; jump discontinuity
- Not removable

7) $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ 1-x & \text{if } x \geq 1 \end{cases}$



- Not continuous; jump discontinuity
- Not removable

8) $f(x) = \dfrac{x}{|x|-3}$



- Not continuous; vertical asymptotes at $x=3$ and $x=-3$
- Not removable

Find the value of "a" and/or "b" for which the function is continuous.

9) $$f(x)=\begin{cases} 7x-2 & \text{if } x\le 1 \\ ax^2 & \text{if } x>1 \end{cases}$$ $7\cdot 1-2=5$

$ax^2=5$

$a1^2=5$

$\boxed{a=5}$

10) $$f(x)=\begin{cases} ax^2 & \text{if } x\le 2 \\ 2x+a & \text{if } x>2 \end{cases}$$

$ax^2=2x+a \quad x=2$

$a(2)^2=2(2)+a$

$4a=4+a$

$3a=4$

$\boxed{a=\frac{4}{3}}$

11) $$f(x)=\begin{cases} x+1 & \text{if } x<1 \\ ax+b & \text{if } 1\le x<2 \\ 3x & \text{if } x\ge 2 \end{cases}$$

$x+1=1+1=2$

$a(1)+b=2$

$3x=3(2)=6$

$a(2)+b=6$

$a+b=2$

$-2a+b=6$

$-a=-4$

$\boxed{a=4}$

$a+b=2$

$4+b=2$

$\boxed{b=-2}$

Are the following functions continuous at all points in the natural domain? If the function is not continuous, does it have a removable discontinuity? If it has a removable discontinuity, create a continuous function.

12) $f(x)=\dfrac{x^2-16}{x+4}=\dfrac{(x+4)(x-4)}{x+4}$

- Not continuous at $x=-4$
- It is removable since there is a hole at $x=-4$.
- Continuous Function

$$f(x)=\begin{cases}\dfrac{x^2-16}{x+4}, & x\neq -4\\ x-4, & x=-4\end{cases}$$

13) $f(x)=\dfrac{2x^2-x-1}{x-1}=\dfrac{(2x+1)(x-1)}{x-1}$

- Not continuous at $x=1$
- It is removable since there is a hole at $x=1$
- Continuous Function

$$f(x)=\begin{cases}\dfrac{2x^2-x-1}{x-1}, & x\neq 1\\ 2x+1, & x=1\end{cases}$$

14) $f(x)=\dfrac{9x^2-4}{3x+2}=\dfrac{(3x+2)(3x-2)}{3x+2}$

- Not continuous at $x=-\frac{2}{3}$
- It is removable since there is a hole at $x=-\frac{2}{3}$
- Continuous Function

$$f(x)=\begin{cases}\dfrac{9x^2-4}{3x+2}, & x\neq -\frac{2}{3}\\ 3x-2, & x=-\frac{2}{3}\end{cases}$$

15) $g(t)=\dfrac{\sin t}{t}$

- Not continuous at $x=0$
- It is removable since there is a hole at $x=0$
- Continuous Function

$$f(x)=\begin{cases}\dfrac{\sin t}{t}, & x\neq 0\\ 1, & x=0\end{cases}$$