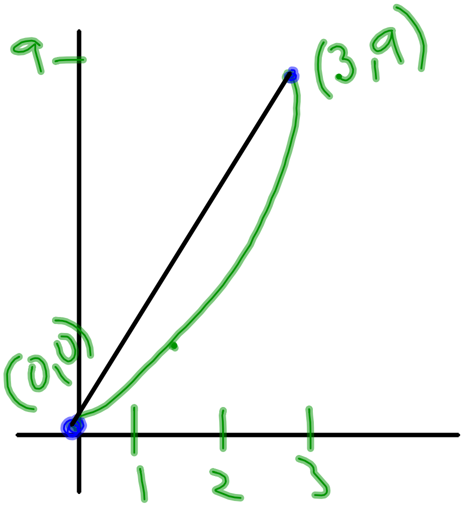
2.4 Rates of Change and Tangent Lines Day 1

Slope of a line = $\frac{\Delta y}{\Delta x}$

average rate of change = $\frac{\text{amount of change}}{\text{time it takes}}$

Ex 1) $y = x^2$ [0, 3]



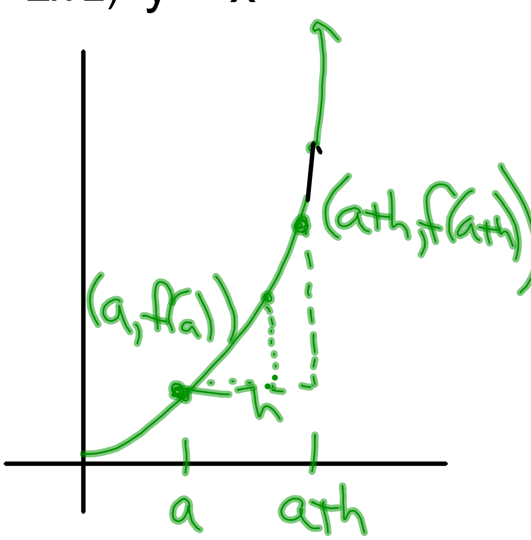
Find the slope of the secant line

$$\frac{\Delta y}{\Delta x} = \frac{9-0}{3-0} = \boxed{3}$$

Slope of a line = $\frac{\Delta y}{\Delta x}$

$y=f(x)$

Ex 2) $y = x^2$



Find the slope at a given point

$$m = \frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{a+h-a}$$

$$= \frac{f(a+h)-f(a)}{h}$$

If we want the slope at a given point, we want $h \to 0$

Thus ⟶

$$\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

- Slope at a given point
- Slope of the tangent line
- Numerical Derivative

Ex 3) Find the slope of $y = x^2+2$ at $x = 1$ $\boxed{a=x}$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2+2 - (1^2+2)}{h} = \lim_{h \to 0} \frac{1+2h+h^2+2-3}{h}$$

$$= \lim_{h \to 0} \frac{h^2+2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(h+2)}{\cancel{h}} = 0+2 = 2$$

Ex4) Find an equation for the line tangent to the graph of $y = x^2+2$ at $x = 1$

$y = 1^2 + 2$

$(1, 3)$
$x \quad y$

*from example $m = 2$.

$y = mx + b$

$\boxed{y = 2x + 1}$

$3 = 2(1) + b$

$1 = b$

Note: Normal means $\perp$