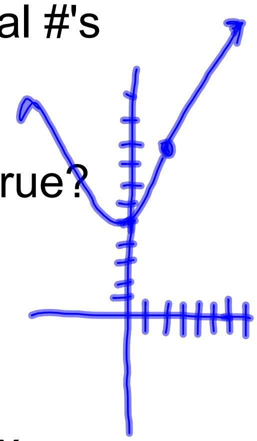
2.4 Rates of Change and Tangent Lines Day 2

Ex 1) If $f(x) = \begin{cases} x^2 + 5 & x < 2 \\ 7x - 5 & x \geq 2 \end{cases}$ for all real #'s

then which of the following must be true?



A. f(x) is continuous everywhere. True
B. f(x) is continuous everywhere except $x = 2$. False
C. f(x) is continuous everywhere except $x = -2$ and 2. False

Ex 2) If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$

then $f(4) = ?$

A. 1
B. 8/7
C. -1
D. 0
E. undefined

$$\frac{(x-3)\cancel{(x-4)}}{\cancel{x-4}}$$

$y = x-3$
$y = 4-3 = 1$

Ex 3)

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$$

$\frac{0}{0}$

$= \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}$

A. 0
B. 10
C. -10
D. 5
E. Does not exist

$\lim_{x \to 5} x+5 = 10$

$x=5$ hole

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- Slope at a given point
- Slope of the tangent line
- Numerical Derivative

Ex 4) $f(x) = x^2 - 4x$ Find the slope at x = 1

$$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2 - 4(1+h) - (1^2 - 4(1))}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + 2h + h^2 - \cancel{4} - 4h + \cancel{3}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2+h-4)}{\cancel{h}} = \lim_{h\to 0} h-2 = 0-2 = \boxed{-2}$$

Ex 5) $f(x) = \frac{1}{x-3}$ Find the slope at x = 4

$$\lim_{h\to 0} \frac{f(4+h)-f(4)}{h} = \lim_{h\to 0} \frac{\frac{1}{(4+h)-3} - \left(\frac{1}{4-3}\right)}{h}$$

$$\lim_{h\to 0} \frac{\frac{1}{h+1} - 1\frac{h+1}{h+1}}{h} = \lim_{h\to 0} \frac{\frac{1}{h+1} - \frac{h+1}{h+1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{h+1}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-\cancel{h}}{h+1} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{h+1} = \frac{-1}{0+1} = -1$$

$\sqrt{4}=2$ $(4,2)$

Ex 6) $f(x) = \sqrt{x}$ Find the slope at x = 4

Then write an equation for the tangent line and normal line.

FOIL

$$\lim_{h\to 0}\frac{f(4+h)-f(4)}{h} = \lim_{h\to 0}\left(\frac{\sqrt{4+h}-\overset{2}{\cancel{\sqrt{4}}}}{h}\right)\left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$$

$$\lim_{h\to 0}\frac{4+h-4}{h(\sqrt{4+h}+2)} = \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2} = \frac{1}{4} \leftarrow m$$

Tangent

$m = \frac{1}{4}$ $(4,2)$

$\boxed{y = \frac{1}{4}x + 1}$

$2 = \frac{1}{4}\cdot 4 + b$

$1 = b$

Normal $\perp$

$m = -4$ $(4,2)$

$\boxed{y = -4x + 18}$

$2 = -4(4) + b$

$18 = b$