AP Calculus
2.4 Worksheet

SKIP 7, 8

Key

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is a difference quotient?

it gives an approximation of the derivative for a given value

$\frac{f(a+h)-f(a)}{h}$

2. How do you find the slope of a curve (aka slope of the tangent line to a curve) when $x = a$?

$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$.

3. What is a *normal line*? The normal line to a curve at a point is the line perpendicular to the tangent at a point.

4. What is the difference between the AVERAGE RATE OF CHANGE and INSTANTANEOUS RATE OF CHANGE?

it's the amount of change divided by the time it takes.

it's the instantaneous speed at any time t (is slope! at a given point)

5. Let $f(x) = x^3$.

$(a+h)^3 = (a^2+2ah+h^2)(a+h) = a^3 + 2a^2h + ah^2 + a^2h + 2ah^2 + h^3$

a) Find the slope of the curve at $x = a$.

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^3 - a^3}{h} = \lim_{h\to 0} \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h}$$

$$= \lim_{h\to 0} \frac{h(3a^2+3ah+h^2)}{h} = \lim_{h\to 0} 3a^2 + 3ah + h^2 = \boxed{3a^2} = \text{slope}$$

b) When does the slope equal 12?

$3a^2 = 12$

$a^2 = 4$

$\boxed{a = \pm 2}$

c) Write the equation of the tangent line to the curve at $x = 4$.

$m = 3a^2 = 3(4)^2 = 48$

$(4, 4^3)$

$(4, 64)$

$\boxed{y - 64 = 48(x-4)}$

d) Write the equation of the normal line to the curve at $x = 4$.

$(4, 64)$

$m = -\frac{1}{48}$

$\boxed{y - 64 = -\frac{1}{48}(x-4)}$

6. Let $g(x)=\sqrt{x}$

a) Find the average rate of change from $x = 4$ to $x = 9$.

$$\frac{f(a)-f(a)}{9-4} = \frac{\sqrt{9}-\sqrt{4}}{9-4} = \frac{3-2}{5} = \boxed{\frac{1}{5}}$$

b) Find the instantaneous rate of change at $x = 9$.

$a=9$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{f(9+h)-f(a)}{h} = \lim_{h\to 0}\frac{\sqrt{9+h}-\sqrt{9}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}\cdot\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \lim_{h\to 0}\frac{9+h-9}{h(\sqrt{9+h}+3)} = \lim_{h\to 0}\frac{h^{1}}{h(\sqrt{9+h}+3)} = \lim_{h\to 0}\frac{1}{\sqrt{9+h}+3}$$

$$= \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

c) Write the equation of the tangent line when $x = 9$, $y = \sqrt{9}$ or 3

$$\boxed{y-3=\frac{1}{6}(x-9)}$$

d) Write the equation of the normal line when $x = 9$.

$$\boxed{y-3=-6(x-9)}$$

~~7. Complete the following from the textbook: page 92 – 93 #3, 6, 7, 11, 12, 23, 25, 29, 38, and 39~~

8. You should also begin reviewing for your chapter 2 test: page 95 – 97 #1 – 29, 31, 33, 35, 39, 40, 43 – 49, 52. This isn't due until the day of your exam.