3.1 Derivative of a Function

Slope

rate of change

Definition of Derivative

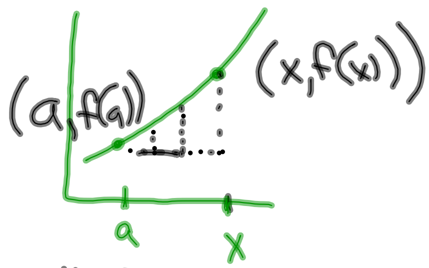
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

alternate form

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Notation for finding the derivative:

$f'(x)$, y', $\frac{dy}{dx}$, $\frac{d}{dx}$, $\frac{df}{dx}$, $\frac{d}{dx}f(x)$



$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Ex 1) Find $f'(x)$ if $f(x) = x^2 + 4$ at $x = 1$.

$a = 1$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$\lim_{h \to 0} \frac{(1+h)^2 + 4 - (1^2+4)}{h}$$

$$\lim_{h \to 0} \frac{h^2 + 2h + \cancel{5} - \cancel{5}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(h+2)}{\cancel{h}} = 0 + 2 = 2$$

$m = 2$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$\lim_{x \to 1} \frac{x^2 + 4 - (1^2+4)}{x - 1}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$= 1 + 1 = 2$$

Ex 2) Find y' for f(x) = $x^2 - 1$ at a = -2.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

Ex 3) Find $\frac{dy}{dx}$ of f(x) = √(x + 2).

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$\lim_{h\to 0} \frac{x+h+2-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$\lim_{h\to 0} \frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$= \frac{1}{\sqrt{x+0+2}+\sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

dy/dx

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a} \cdot \frac{\sqrt{x+2}+\sqrt{a+2}}{\sqrt{x+2}+\sqrt{a+2}}$$

$$\lim_{x\to a} \frac{x+2-a-2}{(x-a)(\sqrt{x+2}+\sqrt{a+2})}$$

$$\lim_{x\to a} \frac{\cancel{x-a}\ 1}{\cancel{(x-a)}(\sqrt{x+2}+\sqrt{a+2})}$$

$$= \frac{1}{\sqrt{a+2}+\sqrt{a+2}} = \boxed{\frac{1}{2\sqrt{a+2}}} = dy/dx$$

Ex 4) Use the definition of the derivative to find f'(1) for $f(x) = \frac{1}{x^2}$.

$x=1$

$$\lim_{h\to0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to0}\frac{\frac{1}{(1+h)^2}-1}{h}$$

$$\lim_{h\to0}\frac{\frac{1}{(1+h)^2}-\frac{(1+h)^2}{(1+h)^2}}{h} = \lim_{h\to0}\frac{\frac{\cancel{1-1}-2h-h^2}{(1+h)^2}}{h} =$$

$$=\lim_{h\to0}\frac{\cancel{h}(-2-h)}{(1+h)^2}\cdot\frac{1}{\cancel{h}}$$

$$=\lim_{h\to0}\frac{-2-h}{(1+h)^2} = \frac{-2-0}{(1+0)^2} = -2 = f'(1)$$

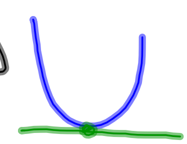
Ex 5) At what point is the tangent to $f(x) = x^2 + 4x - 1$ horizontal?

problem #29 from2.4

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$\boxed{(-2, -5)}$$

$$(-2)^2+4(-2)-1$$



$m=0$

$$\lim_{h\to0}\frac{(x+h)^2+4(x+h)-1-(x^2+4x-1)}{h}$$

$$\lim_{h\to0}\frac{\cancel{x^2}+h^2+2xh+\cancel{4x}+4h\cancel{-1}\cancel{-x^2}\cancel{-4x}\cancel{+1}}{h}$$

$$\lim_{h\to0}\frac{\cancel{h}(h+2x+4)}{\cancel{h}} = 0+2x+4 = \boxed{2x+4=m}$$

$$2x+4=0$$

$$2x=-4$$

$$x=-2$$