3.3 Rules for Differentiation Day 2

Power Rule

Ex 1) $y = x^4 + 3x^3 - 2x^{-2} + 4\sqrt{x}$

$y' = 4x^3 + 9x^2 - 2 \cdot -2x^{-3} + 4x^{1/2}$

$= 4x^3 + 9x^2 + \frac{4}{x^3} + 4 \cdot \frac{1}{2}x^{-1/2}$

$y' = \boxed{4x^3 + 9x + \frac{4}{x^3} + \frac{2}{\sqrt{x}}}$

Product Rule

$\frac{d}{dx}(uv) = uv' + vu'$

$\frac{d}{dx} f(x)g(x) =$

$f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

Ex 2) $\frac{d}{dx} \overset{u}{(x - 1)} \overset{v}{(x^2 - 2)}$

$= (x-1)(2x) + (x^2-2)(1)$

$= 2x^2 - 2x + x^2 - 2$

$= \boxed{3x^2 - 2x - 2}$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Ex 3) $\frac{d}{dx}\left(\frac{\overset{u}{x^2 + 2x - 1}}{\underset{v}{x - 4}}\right) = \frac{(x-4)(2x+2) - (x^2+2x-1)(1)}{(x-4)^2}$

$$= \frac{2x^2 - 8x + \cancel{2x} - 8 - x^2 - \cancel{2x} + 1}{(x-4)^2}$$

$$= \boxed{\frac{x^2 - 8x - 7}{(x-4)^2}} = f'(x) = \frac{dy}{dx}$$

Ex 4) $f(x) = x^4 - x^3 + x^2 - 2x + 6 \quad (1, 5)$

$1 - 1 + 1 - 2 + 6 = 5$

Find equations for the tangent and normal lines at x = 1.

$f'(x) = 4x^3 - 3x^2 + 2x - 2$

$f'(1) = 4(1)^3 - 3(1)^2 + 2(1) - 2 = 1 = m$ slope of tangent line

Tangent

$\boxed{y - 5 = 1(x - 1)}$ $\quad y - y_1 = m(x - x_1)$

Normal $m = -1 \quad (1, 5)$

$\boxed{y - 5 = -1(x - 1)}$

Ex 5) $f(x) = x^3 + 3x^2 - 3x + 6$

- looking for "x"

Where is the tangent line horizontal?

$m=0$

$f'(x) = 3x^2 + 6x - 3$

$0 = 3x^2 + 6x - 3$

$0 = 3(x^2 + 2x - 1)$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{2}}{2} = \frac{-\cancel{2} \pm \cancel{2}\sqrt{2}}{\cancel{2}}$

$x = -1 \pm \sqrt{2}$

Ex 6) Suppose $u(1) = 2$, $u'(1) = 3$, $v(1) = -2$, $v'(1) = 4$

$\frac{d}{dx}(uv)$

$= u \cdot v' + u' \cdot v$

$= 2 \cdot 4 + 3 \cdot -2$

$= 8 + -6 = 2$

$\frac{d}{dx}\, 2u - 4v + 3uv$

$\frac{d}{dx}\frac{u}{v}$

Ex 7) $y = x^4 - x^3 + x^2 - 2x + 6$

$y' =$

$y'' =$

$y''' =$

$y'''' =$

$y''''' =$

39 $y = 2x^3 - 3x^2 - 12x + 20$

$y' = \boxed{0 = 6x^2 - 6x - 12}$

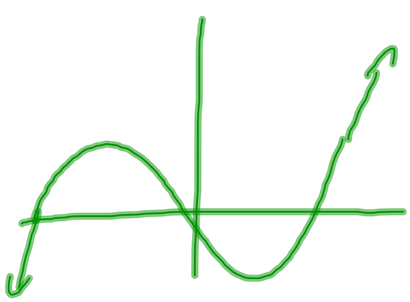
$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x - 2 = 0$ $x + 1 = 0$

$x = 2$ $x = -1$

min max



35 $y = x^{-1} + x^2$

$y' = -1x^{-2} + 2x = \frac{-1}{x^2} + 2x$

$y'' = 2x^{-3} + 2 = \frac{2}{x^3} + 2$

$y''' = -6x^{-4} = \frac{-6}{x^4}$

$y'''' = 24x^{-5} = \frac{24}{x^5}$