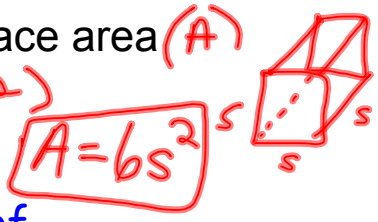
3.4 Velocity and Other Rates Day 2

What does the derivative mean?

- Slope
- How fast something is changing
- Instantaneous rate of change

Ex 1) Write an equation relating surface area (A) of a cube with its side length (s)



Find the instantaneous rate of change for surface area with respect to s.

$A' = 12s$

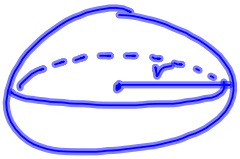
Evaluate A' (1) and A' (2)

$A'(1) = 12(1) = 12$

$A'(2) = 12(2) = 24$

Ex 2) Write an equation relating surface area of a sphere with its radius.

$A = 4\pi r^2$



Find the instantaneous rate of change for surface area with respect to r.

$A' = 8\pi r$

Evaluate A' (1) and A' (2)

$A'(1) = 8\pi(1) = 8\pi$

$A'(2) = 8\pi(2) = 16\pi$

Ex 4) A bullet fired straight up from the moon's surface would reach a height of $s = 832t - 2.6t^2$ after t seconds. How long would it take the bullet to to get back down?

$0 = 832t - 2.6t^2$

$0 = t(832 - 2.6t)$

$t = 0$

$832 - 2.6t = 0$

$832 = 2.6t$

$t = 320$ secs

Ex 5) A particle moves along a line so that its position at time t is given by $s(t) = t^3-6t^2+8t+2$ where s is measured in meters and t is measured in seconds with $t \geq 0$.

Endpoint - Start point

a) Find displacement during first 5 sec.

$\Delta s = s(5) - s(0) = 5^3-6(5)^2+8(5)+2 - (0^3-6(0)^2+8(0)+2)$

$= 17 - 2 = 15$ m

b) Find average velocity during first 5 sec.

$\frac{\Delta s}{\Delta t} = \frac{s(5)-s(0)}{5-0} = \frac{15}{5} = 3$ m/sec

c) When does the particle change direction?

$v=0 \quad v(t) = s'(t) = 3t^2-12t+8 = 0$

d) Where is the particle when s is a minimum?

$t = \frac{12 \pm \sqrt{144-4(3)(8)}}{2(3)}$

$= \frac{12 \pm \sqrt{48}}{6}$

$= \frac{12 \pm 4\sqrt{3}}{6}$

$= \frac{6 \pm 2\sqrt{3}}{3}$

$= \frac{3 \pm \sqrt{3}}{3}$

$2+\frac{2\sqrt{3}}{3} = 3.15$

$2-\frac{2\sqrt{3}}{3} = .86$ max

$s(0) = 2$

smaller than that

$(3.15, -1.08)$

$(3.15)^3-6(3.15)^2+8(3.15)+2$

$*(3.15, -1.08)*$

Ex 6) A body's velocity at time t sec is

$$v = 2t^3 - 9t^2 + 12t - 5.$$

Find the body's speed each time the acceleration is zero

$|v(t)|$

$a(t) = v'(t) = 6t^2-18t+12 = 0$

$6(t^2-3t+2) = 0$

$6(t-1)(t-2) = 0$

$t = 1 \quad t = 2$

$v(1) = 2(1)^3-9(1)^2+12(1)-5 = 0 \quad |0| = 0$ units/sec

$v(2) = 2(2)^3-9(2)^2+12(2)-5 = -1 \quad |-1| = 1$ unit/sec