3.5 Derivatives of Trigonometric Functions

MEMORIZE

f(x)	f'(x)
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \sec^2 x$
$y = \sec x$	$y' = \sec x \tan x$
$y = \csc x$	$y' = -\csc x \cot x$
$y = \cot x$	$y' = -\csc^2 x$

Find y'

Ex 1) $y = \sin x - \tan x + 5x$

$y' = \cos x - \sec^2 x + 5$

Ex 2) $y = x \csc x$ $\quad u \cdot v \quad u \cdot v' + v \cdot u'$

$y' = x \cdot -\csc x \cot x + \csc x \cdot 1$

OR

$y' = \csc x (-x \cot x + 1)$

Find y'

$\frac{v \cdot u' - u \cdot v'}{v^2}$

Ex 3) $y = \frac{\sin x + \cos x}{\cos x}$ (u over v)

$y' = \frac{\cos x(\cos x - \sin x) - (\sin x + \cos x)(-\sin x)}{\cos^2 x}$

$\cos(x)^2$ $(\cos(x))^2$

$y' = \frac{\cos^2 x - \cos x \sin x + \sin^2 x + \sin x \cos x}{\cos^2 x}$

* $\cos^2 x + \sin^2 x = 1$ Pythagorean identity

$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

Find y'

Ex 4) $y = \frac{\cos x}{1 + \sin x}$ (u over v)

$y' = \frac{(1+\sin x)(-\sin x) - (\cos(x) \cdot \cos(x))}{(1+\sin x)^2}$

$y' = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \frac{-\sin x - 1(\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$

$\sin^2 x + \cos^2 x = 1$

$-1 = -\sin^2 x - \cos^2 x$

$= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-1(\sin x + 1)}{(1+\sin x)(1+\sin x)}$

$= \frac{-1}{1+\sin x}$

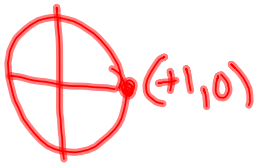
Ex 5) Write an equation for the tangent line and normal line to graph
$y = x + \cos x$ at (0,1)

$x = 0$

$y' = 1 + -\sin x$

$= 1 + -\sin 0 = 1 + 0$

$= 1 = m$



$\sin\theta = y$

$\sin 0 = 0$

Tangent $m = 1$ $(0,1)$

$y - 1 = 1(x - 0)$

Normal $m = -1$ $(0,1)$

$y - 1 = -1(x - 0)$

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23 $\quad s = t^3 - 6t^2 + 9t$

$a(t) = ?$ $\qquad v(t) = 0$

$v = 3t^2 - 12t + 9$

$a = 6t - 12 \rightarrow a(1) = 6 \cdot 1 - 12 = -6$

$a(3) = 6 \cdot 3 - 12 = 6$

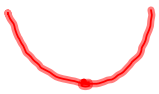
$3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t - 1)(t - 3) = 0$

$t = 1 \quad t = 3$

(19) $s = t^2 - 3t + 2$ →



Change direction when $s' = 0$

$s' = v = 2t - 3$

$v = 2t - 3$

$0 = 2t - 3$

$\frac{3}{2} = \frac{2t}{2}$

$t = 1.5$