3.5 WKST

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

(1) $y = 4\cos x$

$\boxed{y' = -4\sin x}$

(2) $y = x\sin x$

$y' = (x)(\cos x) + (\sin x)(1)$

$\boxed{y' = x\cos x + \sin x}$

(3) $y = \sin x \cos x$

$y' = \sin x(-\sin x) + \cos x(\cos x)$

$y' = -\sin^2 x + \cos^2 x$

$y' = \cos^2 x - \sin^2 x$

$\boxed{y' = \cos 2x}$

(4) $y = \cot x \csc x$

$y' = \cot x(-\csc x \cot x) + \csc x(-\csc^2 x)$

$y' = -\csc x \cot^2 x - \csc^3 x$

$\boxed{y' = -\csc x(\cot^2 + \csc^2 x)}$ ⟶ OPTIONAL STEP

(5) $y = \sin x \sec x$

$y' = (\sin x)(\sec x \tan x) + (\sec x)(\cos x)$

$y' = \sin\left(\frac{1}{\cos x}\right)\tan x + \left(\frac{1}{\cos x}\right)(\cos x)$

$y' = \left(\frac{\sin x}{\cos x}\right)(\tan x) + \left(\frac{\cos x}{\cos x}\right)$

$y' = (\tan x)(\tan x) + 1 = \tan^2 x + 1 = \boxed{\sec^2 x}$

$y = \sin x \sec x$

$y = \sin x\left(\frac{1}{\cos x}\right) = \frac{\sin x}{\cos x}$

$y = \tan x$

$\boxed{y' = \sec^2 x}$

(6) $y = \cos x\,(x - \cot x)$

$y' = \cos x\,(1 - (-\csc^2 x)) + (x - \cot x)(-\sin x)$

$y' = \cos x + \cos x \csc^2 x - x\sin x + \sin x \cot x$

$y' = \cos x + \cos x \csc^2 x - x\sin x + \sin x\left(\frac{\cos x}{\sin x}\right)$

$\boxed{y' = 2\cos x + \cos x \csc^2 x - x\sin x}$

(7) $y = \sec x \tan x$

$y' = (\sec x)(\sec^2 x) + \tan x\,(\sec x \tan x)$

$y' = \sec^3 x + \sec x \tan^2 x$

$\boxed{y' = \sec x\,(\sec^2 x + \tan^2 x)}$

(8) $y = \csc^2 x \cot x$

$y' = \csc^2 x\,(-\csc^2 x) + \cot x\,[(2\csc x)\cdot(-\csc x \cot x)]$

$y' = -\csc^4 x + \cot x\,(-2\csc^2 x \cot x)$

$y' = -\csc^4 x - 2\csc^2 x \cot^2 x$

$\boxed{y' = -\csc^2 x\,(\csc^2 x + 2\cot^2 x)}$

(9) $y = \dfrac{x}{2 + \sin x}$

$y' = \dfrac{(2 + \sin x)(1) - (x)(\cos x)}{(2 + \sin x)^2}$

$\boxed{y' = \dfrac{2 + \sin x - x\cos x}{(2 + \sin x)^2}}$

(10) $y = \dfrac{\tan x}{1+\sin x}$

$y' = \dfrac{(1+\sin x)(\sec^2 x) - (\tan x)(+\cos x)}{(1+\sin x)^2}$

$\boxed{y' = \dfrac{\sec^2 x + \sin x \sec^2 x + \cos x \tan x}{(1+\sin x)^2}} = \boxed{\dfrac{\sec^2 x + \sin x \sec^2 x - \sin x}{(1+\sin x)^2}}$

(11) $y = \dfrac{x^2 + 4\cot x}{x + \tan x}$

$y' = \dfrac{(x+\tan x)(2x - 4\csc^2 x) - (x^2 + 4\cot x)(1+\sec^2 x)}{(x+\tan x)^2}$

$y' = 2x^2 - 4x\csc^2 x + 2x\tan x - 4\tan x \csc^2 x - \left[x^2 + x^2\sec^2 x + 4\cot x + 4\cot x \sec^2 x\right]$

$= \dfrac{x^2 - 4x\csc^2 x + 2x\tan x - 4\tan x\csc^2 x - x^2\sec^2 x - 4\cot x - 4\cot x \sec^2 x}{(x+\tan x)^2}$

(12) $y = x\csc x - \dfrac{x}{\cot x} = x\csc x - x\tan x$

$y' = x(-\csc x \cot x) + \csc x(1) - \left[\dfrac{\cot x(1) - x(-\csc^2 x)}{\cot^2 x}\right]$

$y' = -x\csc x\cot x + \csc x - \dfrac{\cot x - x\csc^2 x}{\cot^2 x}$

(13) $y = \tan x$ at $\left(\frac{\pi}{4}, 1\right)$

$y' = \sec^2 x$

$y'(\pi/4) = \left(\sec \frac{\pi}{4}\right)^2$

$y'\left(\frac{\pi}{4}\right) = 2$

$y = 2\left(x - \frac{\pi}{4}\right) + 1$

$\boxed{y = 2x - \frac{\pi}{2} + 1}$

(14) $y = x \sin x$ at $(\pi, 1)$

From #2 $y' = x\cos x + \sin x$

$y'(\pi) = \pi \cos\pi + \sin\pi$

$y'(\pi) = -\pi$

$y = -\pi(x - \pi) + 1$

$\boxed{y = -\pi x + \pi^2 + 1}$