3.6 Chain Rule

Order of Operations

Ex 1) $y = (3x + 1)^2$

Evaluate at x = 1

$y = (3\cdot 1 + 1)^2$

$= 4^2 = 16$

inside to outside

Ex 2) $y = \cos(2x + \pi)$

Evaluate at $x = \pi/2$

$y = \cos(2\cdot\frac{\pi}{2} + \pi)$

$= \cos 2\pi$

$= 1$

Ex 3) $y = (3x + 1)^2$

"Outside to inside"

Find $\frac{dy}{dx}$

$y' = 2(3x+1)^1 \cdot 3$

$= 6(3x+1) = \boxed{18x+6}$

or $y = (3x+1)(3x+1)$

$y' = (3x+1)3 + (3x+1)3$

$= 9x+3 + 9x+3$

$= \boxed{18x+6}$

or $y = 9x^2 + 6x + 1$

$\boxed{y' = 18x + 6}$

Ex 4) $y = (3x^2 + 6x)^5$

Find $\frac{dy}{dx}$.

$y' = \boxed{5(3x^2+6x)^4 \cdot (6x+6)}$

Ex 5) $y = (3x + 1)^2$

Find $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

let $u = 3x+1$

$\frac{du}{dx} = 3$

$y = u^2$

$\frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 3$

$= 6u$

$= 6(3x+1)$

$\boxed{= 18x+6}$

Ex 6) $y = (3x^2 + 6x)^5$

Find $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

let $u = 3x^2+6x$

$\frac{du}{dx} = 6x+6$

$y = u^5$

$\frac{dy}{du} = 5u^4$

$\frac{dy}{dx} = 5u^4(6x+6)$

$= \boxed{5(3x^2+6x)^4(6x+6)}$

Ex 7) $y = (x^2 + 2x + 3)^3$

Find $\frac{dy}{dx}$.

$y' = \boxed{3(x^2+2x+3)^2 \cdot (2x+2)}$

Ex 8) $y = \tan(5x)$

Find $\frac{dy}{dx}$

$y' = \sec^2(5x) \cdot 5$

$= \boxed{5\sec^2(5x)}$

Ex 9) y = sin $(x^2 + 4)$

Find $\frac{dy}{dx}$

$y' = \cos(x^2+4) \cdot 2x$

$= \boxed{2x\cos(x^2+4)}$

Ex 10) $y = \cos(2x + 3)^3$

Find $\frac{dy}{dx}$

$y = \cos^3(2x+3)$

$* \; y = (\cos(2x+3))^3$

$y' = 3(\cos(2x+3))^2(-\sin(2x+3)) \cdot 2$

$y' = -6(\cos(2x+3))^2\sin(2x+3)$

or

$-6\cos^2(2x+3)\sin(2x+3)$

Ex 11) $y = 2 \sin \sqrt{x^2 - 9}$

$(x^2-9)^{1/2}$

Find $\frac{dy}{dx}$ $= (\cancel{2}\cos\sqrt{x^2-9}) \cdot \frac{1}{\cancel{2}}(x^2-9)^{-1/2} \cdot 2x$

$= \boxed{\dfrac{2x\cos\sqrt{x^2-9}}{\sqrt{x^2-9}}}$

Ex 12) $y = \sin^3 x \cdot \tan(4x)$

$u \cdot v \quad = u \cdot v' + v \cdot u'$

Find $\frac{dy}{dx}$ $= \boxed{\sin^3 x \cdot \sec^2(4x) \cdot 4 + \tan(4x) \cdot 3(\sin x)^2 \cdot \cos x}$

*Note

$\sin^3 x$

* $(\sin x)^3$

Ex 13) $y = \dfrac{x}{\sqrt{1+x^2}}$

$u = x$, $v = \sqrt{1+x^2}$ $\quad (1+x^2)^{1/2} \quad \dfrac{v \cdot u' - u \cdot v'}{v^2}$

Find $\frac{dy}{dx}$

$$y' = \frac{\sqrt{1+x^2} \cdot 1 - x \cdot \cancel{\tfrac{1}{2}}(1+x^2)^{-1/2} \cdot \cancel{2}x}{(\sqrt{1+x^2})^2} = \frac{\sqrt{1+x^2} - \dfrac{x^2}{\sqrt{1+x^2}}}{(\sqrt{1+x^2})^2 \cdot \frac{1}{1}}$$

$$= \frac{\sqrt{1+x^2}}{(\sqrt{1+x^2})^2} - \frac{x^2}{\sqrt{1+x^2}} \cdot \frac{1}{(\sqrt{1+x^2})^2}$$

$$= \frac{1}{\sqrt{1+x^2}} - \frac{x^2}{(\sqrt{1+x^2})^3} = \frac{\overset{1+x^2}{(\sqrt{1+x^2})^2}}{(\sqrt{1+x^2})^3} - \frac{x^2}{(\sqrt{1+x^2})^3}$$

$$= \frac{1}{(\sqrt{1+x^2})^3}$$

Ex 14) $y = (1 + \cos^2(3x))^5$

$(\cos(3x))^2$

Find $\frac{dy}{dx} = 5(1+\cos^2(3x))^4 (2\cos(3x))^1 \cdot -\sin(3x) \cdot 3$

$= -30(1+\cos^2(3x))^4 \cdot \cos(3x) \cdot \sin(3x)$