3.6 The Chain Rule Day 2

Ex 1) Let $f(u) = u^3 + u$ Find $(f \circ g)'$

$u = g(x) = 4x$

$(f \circ g)(x) = f(g(x)) = f(4x) = (4x)^3 + 4x$
$= 64x^3 + 4x$

$(f \circ g)'(x) = 64 \cdot 3x^2 + 4 = 192x^2 + 4$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$f(u) = u^3 + u$
$f'u = 3u^2 + 1$

$g(x) = 4x = u$
$g'(x) = 4 = \frac{du}{dx}$

$= (3u^2 + 1)(4)$
$= 12u^2 + 4$
$= 12(4x)^2 + 4$
$= 12 \cdot 16 \cdot x^2 + 4 = 192x^2 + 4$

Ex 2) Given

$f(1) = 2$ $f'(1) = 3$ $f'(2) = -4$

$g(1) = 2$ $g'(1) = -3$ $g'(2) = 5$

If $h(x) = f(g(x))$

Find $h'(1) = (f \circ g)'(x) = f(g(x))' = f'(g(x)) \cdot g'(x)$

$x = 1$

$= f'(g(1)) \cdot g'(1)$
$= f'(2) \cdot -3$
$= -4 \cdot -3 = 12$

Ex 3) $x = 3\cos(2t)$

$y = 2\sin(3t)$

Find $\frac{dy}{dx}$ $t = \pi/3$

$$\frac{dx}{dt} = -3\sin(2t)\cdot 2 = -6\sin(2t)$$

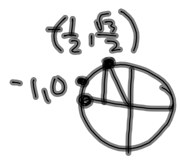
$$\frac{dy}{dt} = 2\cos(3t)\cdot 3 = 6\cos(3t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{6\cos(3t)}{1}\cdot\frac{1}{-6\sin(2t)}$$

$$= \frac{-\cos(3t)}{\sin(2t)} = \frac{-\cos(3\cdot\frac{\pi}{3})}{\sin(2\cdot\frac{\pi}{3})}$$

$$= \frac{-\cos\pi}{\sin\frac{2\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$



Ex 4) $x = 3t^2 + 2$

$y = t^3$

Find $\frac{dy}{dx}$ $t = 1$

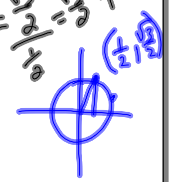
$$\frac{dx}{dt} = 6t$$

$$\frac{dy}{dt} = 3t^2$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{3t^2}{1}\cdot\frac{1}{6t} = \frac{t}{2} = \frac{1}{2}$$

1. $\frac{d}{dx}\sin^2(x^3) = \frac{d}{dx}(\sin(x^3))^2$

$\frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$



$= 2\sin(x^3) \cdot \cos(x^3) \cdot 3x^2$

$= 6x^2 \sin(x^3)(\cos(x^3))$

2. f(x) = sec (2x). Find f '(π/6) $\frac{\pi}{3}$

$f'(x) = 2\sec(2x)\tan(2x) = 2\sec(2\cdot\frac{\pi}{6})\tan(2\cdot\frac{\pi}{6})$

$= 2 \cdot 2 \cdot \sqrt{3} = 4\sqrt{3}$

3. Write an equation for the tangent to the graph of y = x(1 - 2x)² at (1, 1)

A. y=2x+1 B. y= -4x+5

C. y = -2x - 2 D. y = 5x-4

$y' = x \cdot 2(1-2x)^1 \cdot -2 + (1-2x)^2 \cdot 1$

$= 1 \cdot 2(1-2\cdot1) \cdot -2 + (1-2\cdot1)^2$

$= 2(-1)(-2) + 1$

$= 4 + 1 = 5 = m$

4. $y = (1 + \cos^2(7x))^3 = (1 + (\cos(7x))^2)^3$

$y' = 3(1 + (\cos(7x))^2)^2 \cdot 2\cos(7x) \cdot -\sin(7x) \cdot 7$