3.7 Implicit Differentiation

Another technique used you have more than 1 'y'

1. Differentiate both sides with respect to x.
2. Get all terms with dy/dx to one side of the equation.
3. Factor out dy/dx. (if needed)
4. Solve for dy/dx

Ex 1) $y = x^2$ Find $\frac{dy}{dx}$

$\frac{d}{dx}(y) = \frac{d}{dx}(x^2)$

$\frac{dy}{dx} = 2x$

Any time you differentiate a 'y' you must multiply by a dy/dx

Ex 2) $x = y^2$ Find $\frac{dy}{dx}$

$\frac{d}{dx}(x) = \frac{d}{dx}(y^2)$

$1 = 2y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2y}$

$y = \sqrt{x}$

$y = -\sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ $\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$

Ex 3) $2x^3 + 5y^2 = 10$ Find $\frac{dy}{dx}$

$6x^2 + 10y \cdot \frac{dy}{dx} = 0$

$10y\left(\frac{dy}{dx}\right) = -6x^2$

$\frac{dy}{dx} = \frac{-6x^2}{10y} = \frac{-3x^2}{5y}$ *

Ex 4) $x^5 + 4y^3 - 2y^2 = 50$ Find $\frac{dy}{dx}$

$5x^4 + 12y^2 \cdot \frac{dy}{dx} - 4y \cdot \frac{dy}{dx} = 0$

$12y^2 \cdot \frac{dy}{dx} - 4y \cdot \frac{dy}{dx} = -5x^4$

$\frac{dy}{dx}(12y^2 - 4y) = -5x^4$

$\frac{dy}{dx} = \frac{-5x^4}{12y^2 - 4y}$

Ex 5) $x^5 + 4xy^3 - 5y^5 = 4$ Find $\frac{dy}{dx}$

$4xy^3$: $u \cdot v$

$5x^4 + (4 \cdot y^3 + 3y^2 \cdot \frac{dy}{dx} \cdot 4x) - 25y^4 \cdot \frac{dy}{dx} = 0$

$12xy^2 \cdot \frac{dy}{dx} - 25y^4 \cdot \frac{dy}{dx} = -5x^4 - 4y^3$

$\frac{dy}{dx}(12xy^2 - 25y^4) = -5x^4 - 4y^3$

$\frac{dy}{dx} = \frac{-5x^4 - 4y^3}{12xy^2 - 25y^4}$