3.8 Derivatives of Inverse Trigonometric Functions

Day 2

MEMORIZE

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc^{-1}x = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$$

Ex 1) $u=x^3$

$$\frac{d}{dx}\sec^{-1}(x^3) =$$

$$\frac{d}{dx} = \frac{1}{|x^3|\sqrt{(x^3)^2-1}} \cdot 3x^2$$

$$= \frac{3x^2}{|x^3|\sqrt{x^6-1}}$$

$$= \frac{3\cancel{x^2}}{\cancel{x^2}|x|\sqrt{x^6-1}}$$

$$= \boxed{\frac{3}{|x|\sqrt{x^6-1}}}$$

Ex 2) $u=3x$

$$\frac{d}{dx}\cot^{-1}(3x) =$$

$$\frac{d}{dx} = \frac{-1}{1+(3x)^2} \cdot 3$$

$$= \boxed{\frac{-3}{1+9x^2}}$$

Ex 3)

$$\frac{d}{dx}\csc^{-1}\frac{x}{3} =$$

$$= \frac{-1}{\left|\frac{x}{3}\right|\sqrt{\left(\frac{x}{3}\right)^2-1}} \cdot \frac{1}{3}$$

$$= \frac{-\cancel{\frac{1}{3}}}{\cancel{\frac{1}{3}}|x|\sqrt{\frac{x^2}{9}-1}}$$

$$= \frac{-1}{|x|\sqrt{\frac{x^2}{9}-1}}$$

$$= \frac{-1}{|x|\sqrt{\frac{1}{9}(x^2-9)}}$$

$$= \frac{-1}{|x|\frac{1}{3}(\sqrt{x^2-9})} = \frac{-3}{|x|\sqrt{x^2-9}}$$

Ex 4)

$$\frac{d}{dx}\cot^{-1}\sqrt{(x)} = \quad (x)^{1/2}$$

$$= \frac{-1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}(x)^{-1/2}$$

$$= \frac{-1}{2(1+x)\sqrt{x}} = \frac{-1}{2\sqrt{x}(1+x)}$$

$$= \frac{-1}{2\sqrt{x}+2\sqrt{x}\cdot x}$$

$$= \frac{-1}{2\sqrt{x}+2x^{1/2}\cdot x^1} = \boxed{\frac{-1}{2\sqrt{x}+2x^{3/2}}}$$

Ex 5)

$$\frac{d}{dx}\left(\sec^{-1}x + \sqrt{(x^2+1)}\right) = \quad (x^2+1)^{1/2}$$

$$= \frac{1}{|x|\sqrt{x^2-1}} + \cancel{\frac{1}{2}}(x^2+1)^{-1/2}\cdot \cancel{2}x$$

$$= \boxed{\frac{1}{|x|\sqrt{x^2-1}} + \frac{x}{\sqrt{x^2+1}}}$$

Ex 6) Write an equation for the line tangent to $y = \tan^{-1} x$ at $x = 1$

+B

$y' = \frac{1}{1+x^2} = \frac{1}{1+1^2} = \boxed{\frac{1}{2} = m}$ $\boxed{\left(1, \frac{\pi}{4}\right)}$

$\boxed{y - \pi/4 = \frac{1}{2}(x-1)}$

$y = \tan^{-1}(1)$

$\tan y = 1$ $\quad \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$y = \pi/4$

Ex 7) Write an equation for the line tangent to $y = \arcsin x$ at $x = 0.5$

$y = \sin^{-1} x$ $\quad y = \frac{\pi}{6}$ $\quad \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$y' = \frac{1}{\sqrt{1-x^2}}$

$y = \arcsin .5$

$\sin y = .5$

$y = \frac{\pi}{6}$

$= \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{1}{\sqrt{3/4}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = m$

$\boxed{y - \frac{\pi}{6} = \frac{2\sqrt{3}}{3}\left(x - \frac{1}{2}\right)}$

Ex 8) $y = 3x^2 + 4x + 2$

$y(1) = 3(1)^2 + 4(1) + 2 = 9$ $\quad (1, 9)$

$y'(1) = 6x + 4 = 6(1) + 4 = \boxed{10 = m = \frac{\text{Change in } y}{\text{Change in } x}}$

$y^{-1}(9) = 1$ Since $(1,9)$ is a point on "y"... $(9,1)$ is a point on "y^{-1}"

$(y^{-1})'(9) = \frac{1}{10} = \frac{\text{Change in } x}{\text{Change in } y}$

Ex 9) $(2,10)$ $(\cancel{-\frac{5}{3},10})$ $\boxed{\text{Given: } x \geq 0}$ $(10,2)$

If $\boxed{f(x) = 3x^2 - x}$ and $g(x) = f^{-1}(x)$, then $g'(10) =$

$x = 10$

$y = 3x^2 - x$

$\boxed{x = 3y^2 - y}$ $G(x)$

$10 = 3y^2 - y$

$0 = 3y^2 - y - 10$

$= (3y + 5)(y - 2)$

$y = -5/3$ $y = 2$

$(10,2)$

$1 = 6y \cdot \frac{dy}{dx} - 1 \cdot \frac{dy}{dx}$ $G'(x)$

$1 = \frac{dy}{dx}(6y - 1)$

$\frac{dy}{dx} = \frac{1}{6y-1} = \frac{1}{6 \cdot 2 - 1} = \frac{1}{11}$