

3.9 Derivatives of Exponential and Logarithmic Functions
Day 1

$$\log_e u = \ln u$$

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u \cdot \ln e \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \frac{du}{dx}$$

$$\frac{d}{dx} \log_e u = \frac{1}{u \cdot \ln e} \frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

Ex 1) $y = e^{2x}$

$$y' = e^{2x} \cdot 2$$

$$= 2e^{2x}$$

$u \cdot v \quad v' + u \cdot v'$

Ex 2) $y = e^{-5x}$

$$y' = e^{-5x} \cdot -5$$

$$= -5e^{-5x}$$

Ex 3) $y = xe^{2x} + 2x^5$

$$y' = e^{2x} \cdot 1 + x \cdot e^{2x} \cdot 2 + 10x^4$$

$$= e^{2x} + 2xe^{2x} + 10x^4$$

$$= e^{2x}(1 + 2x) + 10x^4$$

Ex 4) $y = x^2 e^{x^3}$

$$y' = e^{x^3} \cdot 2x + x^2 \cdot e^{x^3} \cdot 3x^2$$

$$= 2xe^{x^3} + 3x^4 e^{x^3}$$

$$= xe^{x^3}(2 + 3x^3)$$

Ex 5) $y = 9^{-x}$

$$y' = 9^{-x} \cdot \ln 9 \cdot (-1)$$

$$= \boxed{- (9)^{-x} \ln 9}$$

Ex 6) $y = 3^{\cos x}$

$$y' = 3^{\cos x} \cdot \ln 3 \cdot -\sin x$$

Ex 7) $y = xe^2 - e^x$

$$y' = e^2 - e^x \cdot 1$$

$$= \boxed{e^2 - e^x}$$

Ex 8) $y = \ln x^2$ $a=x^2$

$$y' = \frac{1}{x^2} \cdot 2x$$

$$= \frac{2x}{x^2} = \boxed{\frac{2}{x}}$$

Ex 9) $y = (\ln x)^2$

$$y' = 2(\ln x)' \cdot \frac{1}{x}$$

$$= \boxed{\frac{2(\ln x)}{x}}$$

$u \cdot v \quad u'v + uv'$

Ex 10) $y = \ln \frac{10}{x}$ $\frac{10}{x} = 10x^{-1}$

$$y' = \frac{1}{\frac{10}{x}} \cdot -10x^{-2}$$

$$= \frac{x}{10} \cdot \frac{-10}{x^2} = \boxed{-\frac{1}{x}}$$

Ex 11) $y = x \ln x - x$

$$y' = \ln x \cdot 1 + x \cdot \frac{1}{x} - 1$$

$$y' = \ln x + 1 - 1$$

$$\boxed{y' = \ln x}$$

$$\text{Ex 12) } y = \log_5 \sqrt{x} \quad x^{1/2}$$

$$y' = \frac{1}{\sqrt{x} \cdot \ln 5} \cdot \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{\sqrt{x} \ln 5} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2x \cdot \ln 5}$$

$$\text{Ex 13) } y = \log_3(1 + x \ln 3)$$

$$y' = \frac{1}{(1+x \ln 3) \cdot \ln 3} \cdot \cancel{\ln 3}$$

$$= \boxed{\frac{1}{1+x \ln 3}}$$