3.9 Derivatives of Inverse Trigonometric Functions
Day 2

Ex 1) $y = e^{-x/4}$ $u = -\frac{1}{4}x$

$y' = e^{-x/4} \cdot -\frac{1}{4} = \boxed{\frac{-e^{-x/4}}{4}}$

Ex 2) $y = e^{\sqrt{x}}$ $u = x^{1/2}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

$= \boxed{\frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

Ex 3) $y = \ln(\ln x^2)$

$y' = \frac{1}{\ln x^2} \cdot \frac{1}{x^2} \cdot \frac{2x}{1}$

$\ln 8 = \ln 2^3$

$= \boxed{\frac{2}{x \ln x^2}} = \frac{2}{x \cdot 2 \ln x} = \boxed{\frac{1}{x \ln x}}$

Ex 4) At what point on the graph of $y = 2e^x - 1$ is the tangent line perpendicular to the line $y = -3x + 2$?

$m = -3$

$m = \frac{1}{3}$

(x, y)

$\left(\ln\frac{1}{6}, -\frac{2}{3}\right)$

$y' = 2e^x \rightarrow \frac{1}{2} \cdot 2e^x = \frac{1}{3} \cdot \frac{1}{2} \rightarrow e^x = \frac{1}{6}$

$\ln\frac{1}{6} = x$

$y = 2e^{\ln 1/6} - 1$

$= 2 \cdot \frac{1}{6} - 1$

$= \frac{1}{3} - 1 = -\frac{2}{3}$

Ex 5) A line with slope m passes through the origin and is tangent to y = ln (x/3). What is the value of m?

$y = \ln \frac{x}{3}$

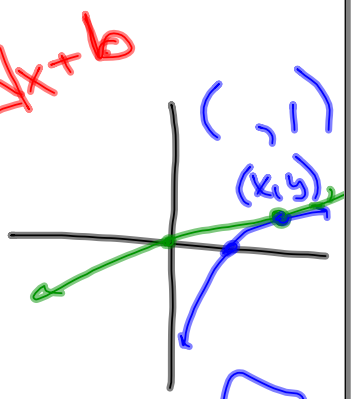
$y' = \frac{1}{\frac{x}{3}} \cdot \frac{1}{3} = \frac{3}{x} \cdot \frac{1}{3} = \frac{1}{x}$

$y = mx + b$

$y = \frac{1}{x} \cdot x + 0$

$y = 1$

$y = mx + b$



$1 = \ln \frac{x}{3}$

$e^1 = \frac{x}{3}$

$3e = x$

$m = \frac{1}{x} = \boxed{\frac{1}{3e}}$

Ex 6) The spread of flu in a certain school is modeled by the equation

$P(t) = \frac{200}{1 + e^{5-t}} = 200(1+e^{5-t})^{-1}$

P = Population
t = days

Estimate the initial number of students with the flu. ($t = 0$)

$P(0) = \frac{200}{1+e^{5-0}} = \frac{200}{(1+e^5)} \approx 1.34 \approx 1 \text{ student}$

How fast is it spreading after 4 days?

rate of change = m

$P'(t) = -200(1+e^{5-t})^{-2} \cdot e^{5-t} \cdot -1$

$= 200(1+e^{5-4})^{-2} e^{5-4}$

$= 200(1+e^1)^{-2} \cdot e^1$

$\approx 39 \text{ students/day}$

Ex 7) Which of the following give the slope of the tangent line to the graph of $y = 2^{1-x}$ at $x = 2$?

a. -1/2
b. 1/2
c. -2
d. 2
e. -(ln 2)/2

$y' = 2^{1-x} \cdot \ln 2 \cdot -1$

$y' = 2^{1-2} \cdot \ln 2 \cdot -1$

$= 2^{-1} \cdot \ln 2 \cdot -1$

$= \boxed{\frac{-\ln 2}{2}}$