4.1 Extreme Values of Functions

Absolute = Global Local = Relative

Extreme Values (Maximums and Minimums)

Occur at: 1. Critical Points OR 2. Endpoints

Critical Points

1. f '(x) = 0
2. f '(x) = undefined

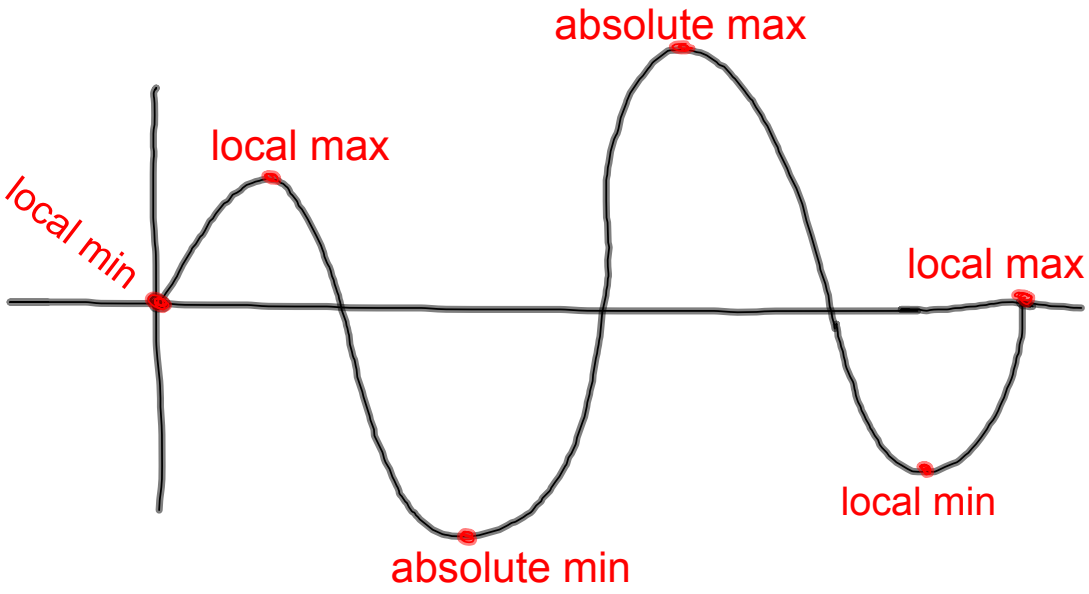
3. Endpoints (May or may not exist)

Absolute Extrema

Highest / Lowest on entire graph

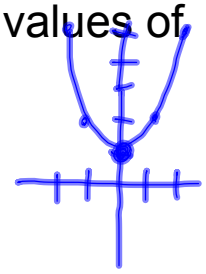
Local Extrema

Highest / Lowest compared to points on each side



Ex 1) $y = x^2 + 1$ $\boxed{-2 \le x \le 2}$

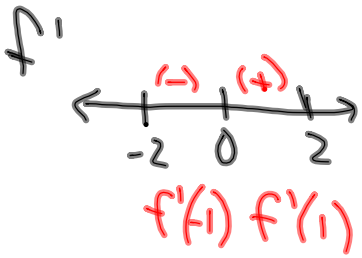
Find the maximum and minimum values of the function.



① $f' = 0$ $y' = 2x$ $2x = 0$ $\boxed{x = 0}$

② f' = undefined $\boxed{\text{Never}}$

③ Endpoints $\boxed{x = -2}$ $\boxed{x = 2}$



$\boxed{\begin{array}{l} x=0 \; y=1 \text{ Abs. Min} \\ x=2 \; y=5 \text{ Abs. Max} \\ x=-2 \; y=5 \text{ Abs. Max} \end{array}}$

$\boxed{y = x^2 + 1}$

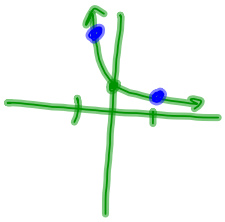
What is the minimum value? (y-value)

$\boxed{y = 1}$

Where does the minimum occur? (x-value)

$\boxed{x = 0}$

Ex 2) $y = e^{-x}$ $\boxed{-1 \le x \le 1}$



Find the maxiumum and minimum values of the function.

① $f' = 0$ $y' = -e^{-x}$ $-e^{-x} = 0$ Never

$e^{-x} = 0$

$\frac{1}{e^x} = 0$

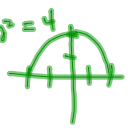
② $f' =$ undefined $y' = \frac{-1}{e^x}$ Never

③ Endpoints $x = -1$ $x = 1$

$x = 1, y = \frac{1}{e}$ Abs Min

$x = -1, y = e$ Abs Max

Ex 3) $y = \sqrt{4 - x^2} = (4 - x^2)^{1/2}$ $(x^2 + y^2 = 4$, $y^2 = 4 - x^2)$



Find the maxiumum and minimum values of the function.

① $f' = 0$ $y' = \frac{1}{2}(4 - x^2)^{-1/2} \cdot -2x$

$= \frac{-x}{\sqrt{4 - x^2}} = 0$ $x = 0$

② $f' =$ undefined $4 - x^2 = 0$ $x = 2, x = -2$

$4 = x^2$

$x = \pm 2$

③ Endpoints

$4 - x^2 \ge 0$

$x^2 \le 4$

$-2 \le x \le 2$

$f'(-1) = +$

$f'(1) = -$

Abs Max

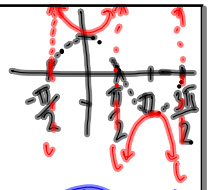
$x = 0, y = 2$

Abs Min

$x = 2$ $y = 0$

$x = -2$ $y = 0$

Ex 4) $y = \sec x$ $\quad y = \frac{1}{\cos x}$ $\quad \boxed{\frac{-\pi}{2} \le x \le \frac{3\pi}{2}}$



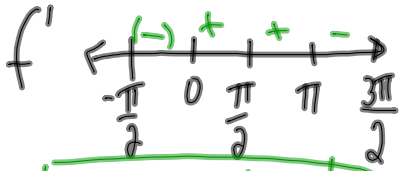
Find the maxiumum and minimum values of the function.

① $f' = 0$ $\quad y' = \sec x \tan x = 0$

$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = 0$ $\quad \sin x = 0$ $\quad x = 0, \pi$

② f' = undefined $\quad \cos x = 0$ $\quad x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

③ Endpoints $\quad x = -\frac{\pi}{2}, \frac{3\pi}{2}$



$x = 0$ min $y = 1$

$x = \pi$ max $y = 1$