4.1 Extreme Values of Functions
Day 2
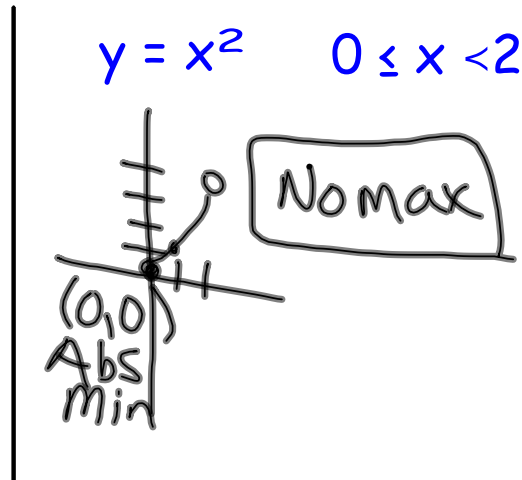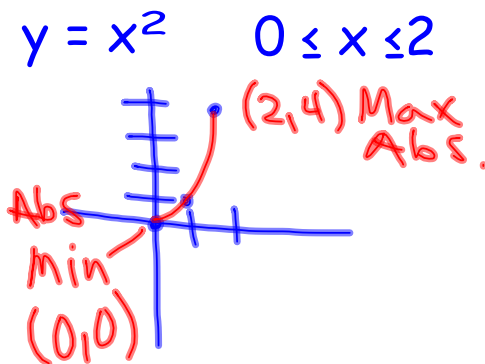
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Extreme Value Theorem $a \leq x \leq b$

If f is <u>continuous</u> on a closed interval [a,b], then f has both a maximum and minimum value.

Ex 1)

$y = x^2$ $0 \leq x \leq 2$

(2,4) Max
Abs.

Abs
Min
(0,0)

$y = x^2$ $0 \leq x < 2$

No max

(0,0)
Abs
min

$a+c$

A local maximum value occurs if and only if f(x) ≤ f(c) for all x <u>in an interval.</u>
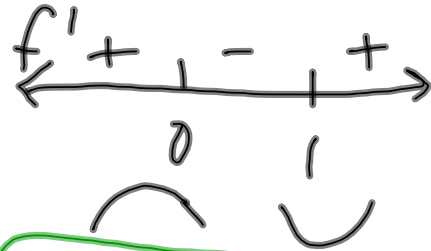
f(c) is bigger than all other y-values

$a+c$

A local minimum value occurs if and only if f(x) ≥ f(c) for all x <u>in an interval.</u>

f(c) is smaller than all other y-values

Ex 2) Find the extreme values of the function.

$$f(x) = \begin{cases} 5 - 2x^2, & x \le 1 \\ x + 2, & x > 1 \end{cases}$$

$5 - 2x^2$

① $f' = 0$
$-4x = 0$
$\boxed{x = 0}$

② $f' = $ undefined
$\boxed{\text{None}}$

③ Endpoints
$\boxed{x = 1}$

$x + 2$

① $f' = 0$
$f' = 1$
$\boxed{\text{Never}}$

② $\boxed{\text{None}}$

③ $\boxed{\cancel{x = 1}}$

$f' \; + \quad - \quad +$

$0 \quad\quad 1$

$\boxed{\begin{array}{l} x = 0, y = 5 \text{ Max} \\ x = 1, y = 3 \text{ Min} \end{array}}$

True or False

If f(c) is a local max of a continuous function f on an open interval (a,b), then f '(c) = 0

False $y = -|x|$

If m is a local minimum and M is a local maximum of a continuous function f on (a,b), then m<M. false

Ex 3) $f(x) = 4x - x^2 + 6$ [0,4]

$f(2) = 4 \cdot 2 - 2^2 + 6 = 10$

Find the absolute maximum value. $(2, 10)$

① $f' = 0$ $f' = 4 - 2x = 0$
$2x = 4$
$\boxed{x = 2}$

or
$\boxed{y = 10}$

② $f' = $ undefined $\boxed{None}$

③ Endpoints $\boxed{x = 0, x = 4}$

f'

(+) (-)
0   2   4

$x = 2$ Max
$y =$

$f' = 0$ or $f' =$ undefined
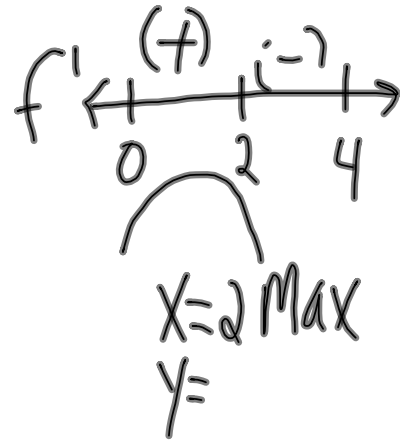
Ex 4) If f is a continuous, decreasing function on [0,10] with a critical point at (4,2), which of the following statements must be false?

a. f(10) is an absolute minimum of f on [0,10] True
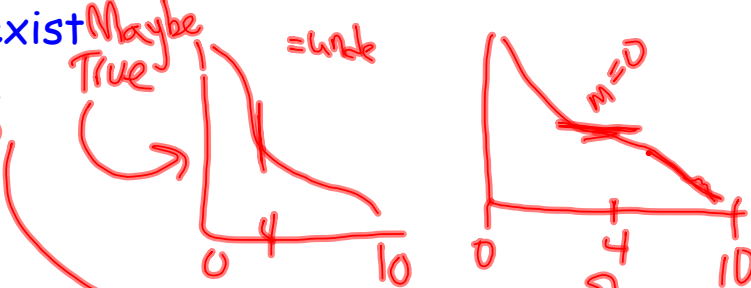b. f(4) is neither a relative max nor min True
c. f'(4) does not exist Maybe True = unk
d. f'(4) = 0 Maybe
e. f'(4) < 0 True

False

Ex 5) Which of the following functions has exactly two local extrema on its domain?

a. y = |x-2|

b. y = x³ - 6x + 5

c. y = x³ + 6x - 5

d. y = tan x

e. y = x + ln x

$f' = und$
$never$

$y' = 3x^2 - 6 = 0$
$3(x^2 - 2) = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

$f' \xleftarrow{+ \quad - \quad +}$
$\quad -\sqrt{2} \quad \sqrt{2}$

$3x^2 + 6 = y'$
$3x^2 + 6 = 0$
$3x^2 = 6$
$x^2 = -2$

$f' = undef$
None
No critical pts
No mins/max

$y' = 1 + \dfrac{1}{x}$
$y' = 0$ No!
$y' = undef.$
yes x=0

$f' \xleftarrow{+} $
$\quad 0$

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reflection over y-axis

Ex 6) If an even function f with domain all real numbers has a local maximum at x = a, then f(-a)...

a. is a local minimum

b. is a local maximum

c. is both a local minimum and a local max

d. could be either a local min or local max

e. is neither a local min nor a local max

odd
Sin

cosx

even

$f(-x) = f(x)$ even
$f(-x) = -f(x)$ odd