Section 4.1 Exercises

1. Minima at $(-2, 0)$ and $(2, 0)$, maximum at $(0, 2)$

3. Maximum at $(0, 5)$ Note that there is no minimum since the endpoint $(2, 0)$ is excluded from the graph.

5. Maximum at $x = b$, minimum at $x = c_2$;
The Extreme Value Theorem applies because f is continuous on $[a, b]$, so both the maximum and minimum exist.

7. Maximum at $x = c$, no minimum;
The Extreme Value Theorem does not apply, because the function is not defined on a closed interval.

9. Maximum at $x = c$, minimum at $x = a$;
The Extreme Value Theorem does not apply, because the function is not continuous.

11. The first derivative $f'(x) = -\frac{1}{x^2} + \frac{1}{x}$ has a zero at $x = 1$.

Critical point value: $f(1) = 1 + \ln 1 = 1$

Endpoint values: $f(0.5) = 2 + \ln 0.5 \approx 1.307$

$$f(4) = \frac{1}{4} + \ln 4 \approx 1.636$$

Maximum value is $\frac{1}{4} + \ln 4$ at $x = 4$;

minimum value is 1 at $x = 1$;

local maximum at $\left(\frac{1}{2}, 2 - \ln 2\right)$

13. The first derivative $h'(x) = \frac{1}{x+1}$ has no zeros, so we need only consider the endpoints.

$h(0) = \ln 1 = 0$ $\quad$ $h(3) = \ln 4$

Maximum value is $\ln 4$ at $x = 3$;

minimum value is 0 at $x = 0$.

15. The first derivative $f'(x) = \cos\left(x + \frac{\pi}{4}\right)$, has zeros at $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$.

Critical point values: $x = \frac{\pi}{4}$ $\quad f(x) = 1$

$x = \frac{5\pi}{4}$ $\quad f(x) = -1$

Endpoint values: $x = 0$ $\quad f(x) = \frac{1}{\sqrt{2}}$

$x = \frac{7\pi}{4}$ $\quad f(x) = 0$

Maximum value is 1 at $x = \frac{\pi}{4}$;

minimum value is -1 at $x = \frac{5\pi}{4}$;

local minimum at $\left(0, \frac{1}{\sqrt{2}}\right)$;

local maximum at $\left(\frac{7\pi}{4}, 0\right)$

17. The first derivative $f'(x) = \frac{2}{5}x^{-3/5}$ is never zero but is undefined at $x = 0$.

Critical point value: $x = 0$ $\quad f(x) = 0$

Endpoint value: $x = -3$ $\quad f(x) = (-3)^{2/5}$

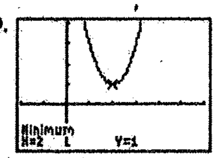
$$= 3^{2/5} \approx 1.552$$

Since $f(x) > 0$ for $x \neq 0$, the critical point at $x = 0$ is a local minimum, and since $f(x) \le (-3)^{2/5}$ for $-3 \le x < 1$, the endpoint value at $x = -3$ is a global maximum.

Maximum value is $3^{2/5}$ at $x = -3$;

minimum value is 0 at $x = 0$.

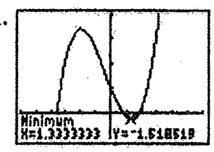
19.



$[-2, 6]$ by $[-2, 4]$

Minimum value is 1 at $x = 2$.

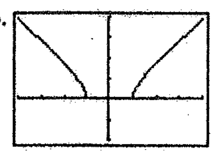
21.



$[-6, 6]$ by $[-5, 20]$

To find the exact values, note that $y' = 3x^2 + 2x - 8 = (3x - 4)(x + 2)$, which is zero when $x = -2$ or $x = \frac{4}{3}$. Local maximum at $(-2, 17)$; local minimum at $\left(\frac{4}{3}, -\frac{41}{27}\right)$

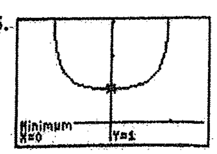
23.



$[-4, 4]$ by $[-2, 4]$

Minimum value is 0 at $x = -1$ and at $x = 1$.

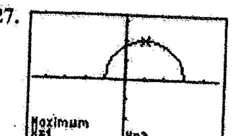
25.



$[-1.5, 1.5]$ by $[-0.5, 3]$

The minimum value is 1 at $x = 0$.

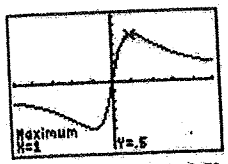
27.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Max of 2 at $x = 1$
Min of 0 at $x = -1 + 3$

29.

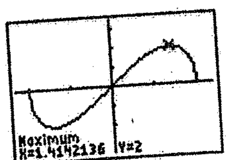


[−5, 5] by [−0.7, 0.7]

Maximum value is $\frac{1}{2}$ at $x = 1$;

minimum value is $-\frac{1}{2}$ at $x = -1$.

37.

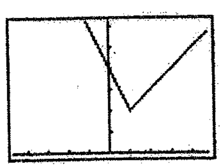


[−2.35, 2.35] by [−3.5, 3.5]

$$y' = x \cdot \frac{1}{2\sqrt{4-x^2}}(-2x) + (1)\sqrt{4-x^2}$$
$$= \frac{-x^2 + (4-x^2)}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}$$

crit. pt.	derivative	extremum	value
$x=-2$	undefined	local max	0
$x=-\sqrt{2}$	0	minimum	−2
$x=\sqrt{2}$	0	maximum	2
$x=2$	undefined	local min	0

39.



[−4.7, 4.7] by [0, 6.2]

$$y' = \begin{cases} -2, & x < 1 \\ 1, & x > 1 \end{cases}$$

crit. pt.	derivative	extremum	value
$x=1$	undefined	minimum	2