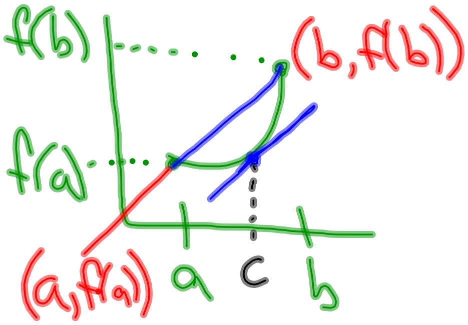
4.2 Mean Value Theorem

$\frac{\Delta y}{\Delta x}$

MVT - Mean Value Theorem

If f(x) is continuous on [a,b] and it is differentiable on (a,b), then there is some point c at which,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Ex 1) Find a point c that satisfies the Mean Value Theorem given the following:

$$y = x^2 \qquad \underset{a}{0} \le x \le \underset{b}{4}$$

① Slope of endpoints

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \frac{f(4) - f(0)}{4 - 0} = \frac{16 - 0}{4} = 4$$

② $y' = 2x$

③ Set equal/solve. $2x = 4$

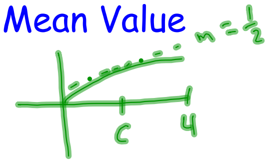
$x = 2$

$\boxed{c = 2}$

Ex 2) Find a point c that satisfies the Mean Value Theorem given the following:

$y = \sqrt{x} \quad 0 \le x \le 4$

a b



① Slope of endpoints $m = \frac{f(4) - f(0)}{4 - 0} = \frac{2-0}{4} = \frac{1}{2}$

② $y' = \frac{1}{2\sqrt{x}}$

③ Set equal | solve. $2 \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2} \cdot 2$

$\frac{1}{\sqrt{x}} = 1$

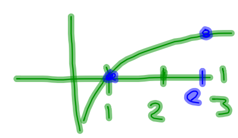
$\sqrt{x} = 1$

$x = 1$

$c = 1$

Ex 3) Find a point c that satisfies the Mean Value Theorem. $y = \ln x \quad 1 \le x \le e$

a b



① Slope between endpoints.

$m = \frac{f(e) - f(1)}{e - 1} = \frac{\ln e - \ln 1}{e - 1} = \frac{1 - 0}{e - 1} = \boxed{\frac{1}{e-1}}$

② find $y' = \frac{1}{x}$

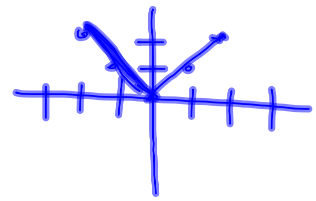
③ Set equal/solve. $\frac{1}{x} = \frac{1}{e-1}$

$x = e - 1$

$\boxed{c = e - 1}$

Ex 4) Find a point c that satisfies the Mean Value Theorem. $y = |x| \quad -2 \le x \le 2$

Cannot Apply MVT Because at $x=0$ is a corner & is therefore non-differentiable.



Ex 5) Find a point c that satisfies the Mean Value Theorem. $y = (\sqrt{x})^3$

$y = x^{3/2} \quad \overset{a}{1} \le x \le \overset{b}{4}$

① Slope between endpoints

$$m = \frac{f(4) - f(1)}{4-1} = \frac{8-1}{4-1} = \boxed{\frac{7}{3}}$$

② Find $y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

③ Set equal/solve.

$$2 \cdot \frac{3\sqrt{x}}{2} = \frac{7}{3} \cdot 2$$

$$\frac{1}{3} \cdot 3\sqrt{x} = \frac{14}{3} \cdot \frac{1}{3}$$

$$(\sqrt{x})^2 = \frac{14^2}{9^2}$$

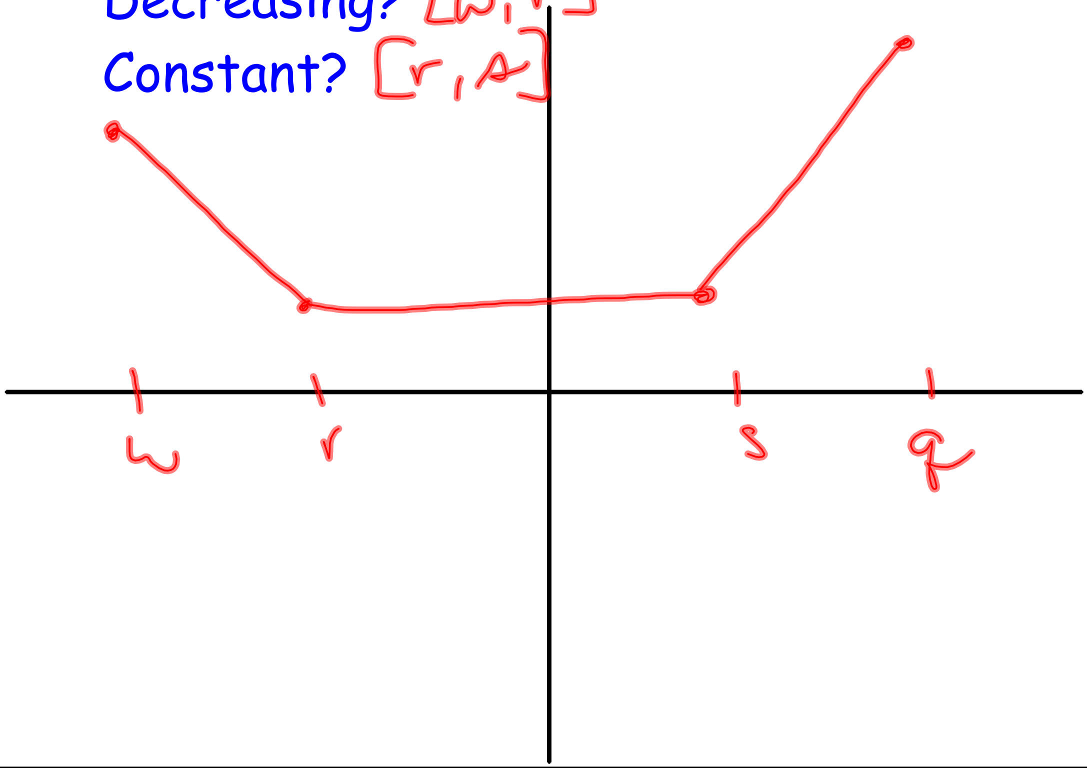
$$x = \frac{196}{81}$$

$$\boxed{c = \frac{196}{81}}$$

Ex 6) Increasing? [s, q]

Decreasing? [w, r]

Constant? [r, s]



Assuming f(x) is continuous and differentiable on [a,b]...

If $f'(x)>0$ for all x on (a,b), then f(x) is increasing on [a,b]

If $f'(x)<0$ for all x on (a,b), then f(x) is decreasing on [a,b]

***A function can be both increasing and decreasing at the same point!

Ex 7) Determine the intervals on which the function is increasing or decreasing.

$f(x) = (x-3)^3$

① $f' = 0$ $\quad 3(x-3)^2 \cdot 1 = f'(x)$

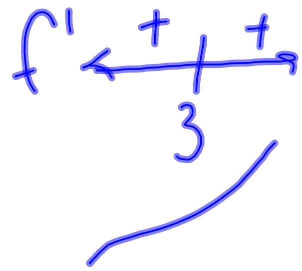
$0 = 3(x-3)^2$

$\sqrt{0} = \sqrt{(x-3)^2}$

$0 = x-3 \quad \boxed{x=3}$

② $f' =$ und None

③ Endpoints - None



$\boxed{\text{increasing}: (-\infty, \infty)}$

Ex 8) Determine the intervals on which the function is increasing or decreasing.

$f(x) = x^4 - 2x^2$

① $f' = 0$ $\quad f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0$

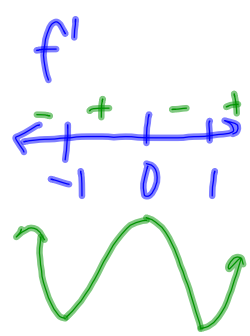
$4x = 0 \qquad x^2 - 1 = 0$

$\boxed{x=0} \qquad x^2 = 1$

$\boxed{x = \pm 1}$

② $f' =$ und None

③ Endpoints: None



increasing: $[-1,0]$, $[1,\infty)$

decreasing: $(-\infty,-1]$, $[0,1]$

Ex 9) Determine the intervals on which the function is increasing or decreasing.

$$f'(x) = \frac{5x-10}{3\sqrt{x}}$$

① $f'=0$ $\quad 0=\frac{5x-10}{3\sqrt{x}}$ $\quad 5x-10=0$

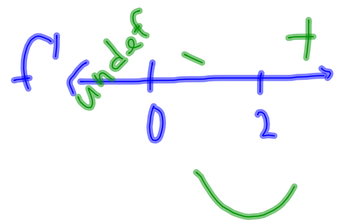
$5x=10$

$\boxed{x=2}$

② $f'=$ undef. $x=0$

③ Endpoints:

$x>0$



increasing:
$[2,\infty)$

decreasing: $(0,2]$