Section 4.2 Exercises

1. (a) Yes.

(b) $f'(x) = \frac{d}{dx}x^2 + 2x - 1 = 2x + 2$

$$2c + 2 = \frac{2-(-1)}{1-0} = 3$$
$$c = \frac{1}{2}.$$

3. (a) No. There is a verticle tangent at $x = 0$.

5. (a) Yes.

(b) $f'(x) = \frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$$\frac{1}{\sqrt{1-c^2}} = \frac{(\pi/2)-(-\pi/2)}{1-(-1)} = \frac{\pi}{2}$$
$$\sqrt{1-c^2} = \frac{2}{\pi}$$
$$c = \sqrt{1-4/\pi^2} \approx 0.771.$$

9. (a) The secant line passes through $(0.5, f(0.5)) = (0.5, 2.5)$ and $(2, f(2)) = (2, 2.5)$, so its equation is $y = 2.5$.

(b) The slope of the secant line is 0, so we need to find c such that $f'(c) = 0$.

$$1 - c^{-2} = 0$$
$$c^{-2} = 1$$
$$c = 1$$
$$f(c) = f(1) = 2$$

The tangent line has slope 0 and passes through $(1, 2)$, so its equation is $y = 2$.

10. (a) The secant line passes through $(1, f(1)) = (1, 0)$ and $(3, f(3)) = (3, \sqrt{2})$, so its slope is

$$\frac{\sqrt{2}-0}{3-1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

The equation is $y = \frac{1}{\sqrt{2}}(x-1) + 0$

or $y = \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}$, or $y \approx 0.707x - 0.707$.

(b) We need to find c such that $f'(c) = \frac{1}{\sqrt{2}}$.

$$\frac{1}{2\sqrt{c-1}} = \frac{1}{\sqrt{2}}$$
$$2\sqrt{c-1} = \sqrt{2}$$
$$c - 1 = \frac{1}{2}$$
$$c = \frac{3}{2}$$
$$f(c) = f\left(\frac{3}{2}\right) = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

The tangent line has slope $\frac{1}{\sqrt{2}}$ and passes through $\left(\frac{3}{2}, \frac{1}{\sqrt{2}}\right)$. Its equation is $y = \frac{1}{\sqrt{2}}\left(x - \frac{3}{2}\right) + \frac{1}{\sqrt{2}}$ or

$y = \frac{1}{\sqrt{2}}x - \frac{1}{2\sqrt{2}}$, or $y \approx 0.707x - 0.354$.

11. Because the trucker's average speed was 79.5 mph, and by then Mean Value Theorem, the trucker must have been going that speed at least once during the trip.

15. (a) $f'(x) = 5 - 2x$

Since $f'(x) > 0$ on $\left(-\infty, \frac{5}{2}\right)$, $f'(x) = 0$ at $x = \frac{5}{2}$, and $f'(x) < 0$ on $\left(\frac{5}{2}, \infty\right)$, we know that $f(x)$ has a local maximum at $x = \frac{5}{2}$. Since $f\left(\frac{5}{2}\right) = \frac{25}{4}$, the local maximum occurs at the point $\left(\frac{5}{2}, \frac{25}{4}\right)$. (This is also a global maximum.)

(b) Since $f'(x) > 0$ on $\left(-\infty, \frac{5}{2}\right)$, $f(x)$ is increasing on $\left(-\infty, \frac{5}{2}\right]$.

(c) Since $f'(x) < 0$ on $\left(\frac{5}{2}, \infty\right)$, $f(x)$ is decreasing on $\left[\frac{5}{2}, \infty\right)$.

17. (a) $h'(x) = -\frac{2}{x^2}$

Since $h'(x)$ is never zero is undefined only where $h(x)$ is undefined, there are no critical points. Also, the domain $(-\infty, 0) \cup (0, \infty)$ has no endpoints. Therefore, $h(x)$ has no local extrema.

(b) Since $h'(x)$ is never positive, $h(x)$ is not increasing on any interval.

(c) Since $h'(x) < 0$ on $(-\infty, 0) \cup (0, \infty)$, $h(x)$ is decreasing on $(-\infty, 0)$ and on $(0, \infty)$.

19. (a) $f'(x) = 2e^{2x}$

Since $f'(x)$ is never zero or undefined, and the domain of $f(x)$ has no endpoints, $f(x)$ has no extrema.

(b) Since $f'(x)$ is always positive, $f(x)$ is increasing on $(-\infty, \infty)$.

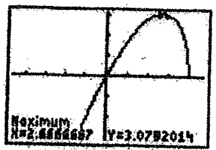
(c) Since $f'(x)$ is never negative, $f(x)$ is not decreasing on any interval.

21. (a) $y' = -\frac{1}{2\sqrt{x+2}}$

In the domain $[-2, \infty)$, y' is never zero and is undefined only at the endpoint $x = -2$. The function y has a local maximum at $(-2, 4)$. (This is also a global maximum.)

(b) Since y' is never positive, y is not increasing on any interval.

(c) Since y' is negative on $(-2, \infty)$, y is decreasing on $[-2, \infty)$.

23. 

$[-4.7, 4.7]$ by $[-3.1, 3.1]$

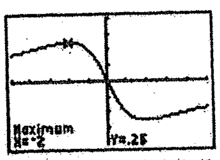
(a) $f'(x) = x \cdot \dfrac{1}{2\sqrt{4-x}}(-1) + \sqrt{4-x}$

$= \dfrac{-3x+8}{2\sqrt{4-x}}$

The local extrema occur at the critical point $x = \dfrac{8}{3}$ and at the endpoint $x = 4$. There is a local (and absolute) maximum at $\left(\dfrac{8}{3}, \dfrac{16}{3\sqrt{3}}\right)$ or approximately $(2.67, 3.08)$, and a local minimum at $(4, 0)$.

(b) Since $f'(x) > 0$ on $\left(-\infty, \dfrac{8}{3}\right)$, $f(x)$ is decreasing on $\left(-\infty, \dfrac{8}{3}\right]$.

(c) Since $f'(x) < 0$ on $\left(\dfrac{8}{3}, 4\right)$, $f(x)$ is decreasing on $\left[\dfrac{8}{3}, 4\right]$.

25. 

$[-5, 5]$ by $[-0.4, 0.4]$

(a) $h'(x) = \dfrac{(x^2+4)(-1)-(-x)(2x)}{(x^2+4)^2} = \dfrac{x^2-4}{(x^2+4)^2}$

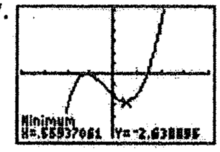
$= \dfrac{(x+2)(x-2)}{(x^2+4)^2}$

The local extrema occur at the critical points, $x = \pm 2$.

There is a local (and absolute) maximum at $\left(-2, \dfrac{1}{4}\right)$ and a local (and absolute) minimum at $\left(2, -\dfrac{1}{4}\right)$.

(b) Since $h'(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$, $h(x)$ is increasing on $(-\infty, -2]$ and $[2, \infty)$.

(c) Since $h'(x) < 0$ on $(-2, 2)$, $h(x)$ is decreasing on $[-2, 2]$.

27. 

$[-4, 4]$ by $[-6, 6]$

(a) $f'(x) = 3x^2 - 2 + 2\sin x$

Note that $3x^2 - 2 > 2$ for $|x| \ge 1.2$ and $|2\sin x| \le 2$ for all x, so $f'(x) > 0$ for $|x| \ge 1.2$. Therefore, all critical points occur in the interval $(-1.2, 1.2)$, as suggested by the graph. Using grapher techniques, there is a local maximum at approximately $(-1.126, -0.036)$, and a local minimum at approximately $(0.559, -2.639)$.

(b) $f(x)$ is increasing on the intervals $(-\infty, -1.126]$ and $[0.559, \infty)$, where the interval endpoints are approximate.

(c) $f(x)$ is decreasing on the interval $[-1.126, 0.559]$, where the interval endpoints are approximate.

43. **(a)** Since $v'(t) = 1.6$, $v(t) = 1.6t + C$. But $v(0) = 0$, so $C = 0$ and $v(t) = 1.6t$. Therefore, $v(30) = 1.6(30) = 48$. The rock will be going 48 m/sec.

(b) Let $s(t)$ represent position.

Since $s'(t) = v(t) = 1.6t$, $s(t) = 0.8t^2 + D$. But $s(0) = 0$, so $D = 0$ and $s(t) = 0.8t^2$. Therefore, $s(30) = 0.8(30)^2 = 720$. The rock travels 720 meters in the 30 seconds it takes to hit bottom, so the bottom of the crevasse is 720 meters below the point of release.

(c) The velocity is now given by $v(t) = 1.6t + C$, where $v(0) = 4$. (Note that the sign of the initial velocity is the same as the sign used for the acceleration, since both act in a downward direction.) Therefore, $v(t) = 1.6t + 4$, and $s(t) = 0.8t^2 + 4t + D$, where $s(0) = 0$ and so $D = 0$. Using $s(t) = 0.8t^2 + 4t$ and the known crevasse depth of 720 meters, we solve $s(t) = 720$ to obtain the positive solution $t \approx 27.604$, and so $v(t) = v(27.604) = 1.6(27.604) + 4 \approx 48.166$. The rock will hit bottom after about 27.604 seconds, and it will be going about 48.166 m/sec.