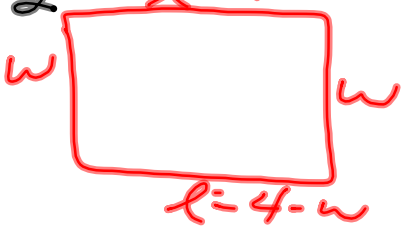
4.4 Modeling and Optimization

Ex 1) Find the maximum area of a rectangle with perimeter = 8.

$A = \ell \cdot w = 2 \cdot 2 = 4$ $2 = \ell = 4 - w$



$A(w) = (4-w)(w)$

$A(x) = 4w - w^2$

$A'(w) = 4 - 2w = 0$

$\frac{4}{2} = \frac{2w}{2}$

$2 = w$

$P = 2\ell + 2w$

$8 = 2\ell + 2w$

$\frac{8 - 2w}{2} = \frac{2\ell}{2}$

$4 - w = \ell$

Ex 2) A rectangular pen is enclosed by fencing. There is also a fence through the middle which separates the pen into 2 equal rectangles. If the total area is 216m², what is the least amount of fencing needed?

minimize the perimeter

$P(x) = 2x + \frac{648}{x}$ $\quad 648x^{-1}$

$P'(x) = 2 + -1 \cdot 648x^{-2}$

$f'=0 \quad 0 = 2 + \frac{-648}{x^2}$

$x^2 \cdot \frac{648}{x^2} = 2x^2$

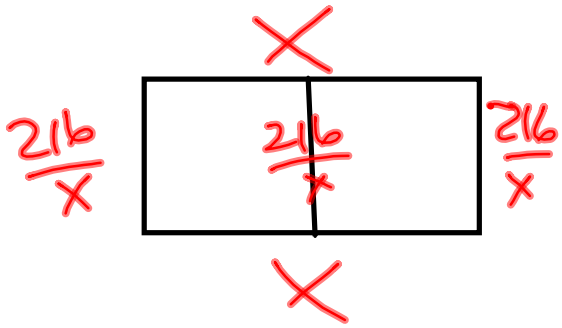
$648 = 2x^2$

$324 = x^2$

$\pm 18 = x$

$18 = x$

$f'=\text{und}$ $\quad x=0$



$A = 216m^2$

$A = xy$

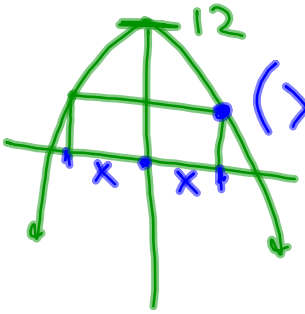
$216 = x \cdot y$

$\frac{216}{x} = y$

$P(x) = 2(18) + \frac{648}{18}$

$= 72m$

Ex 3) A rectangle is positioned so its base is on the x-axis and its other two vertices are on $y = 12-x^2$. Find the maximum area.



$(x, f(x))$

$(x, 12-x^2)$

$A(x) = x \cdot y$

$A(x) = 2x \cdot (12-x^2)$

$A(x) = 24x - 2x^3$

$A'(x) = 24 - 6x^2$

$0 = 24 - 6x^2$

$6x^2 = 24$

$x^2 = 4$

$x = \pm 2$

$A(x) = 2 \cdot 2(12-2^2)$

$= 4(8) = 32 \text{ units}^2$