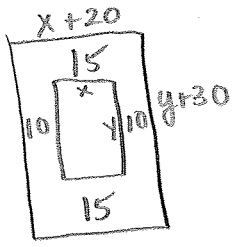
Calculus Quiz Review 4.4-4.6

Name Key
Hour ____

1. The top and bottom margins of a poster are each 15 cm and the side margins are each 10 cm. If the area of printed material on the poster is fixed at 2400 cm², find the dimensions of the poster with the smallest area.



$xy = 2400$

$y = \frac{2400}{x}$

$A = xy$

$A = (x+20)\left(\frac{2400}{x} + 30\right)$

$A = 2400 + \frac{48000}{x} + 30x + 600$

$A' = -\frac{48000}{x^2} + 30 = 0$

$30 = \frac{48000}{x^2}$

$30x^2 = 48000$

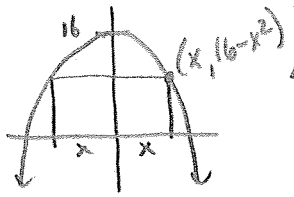
$x^2 = 1600$

$x = 40$

$40 + 20 = 60$ cm

$\frac{2400}{40} + 30 = 90$ cm

2. The base of a rectangle is on the x-axis and its two upper vertices are on the parabola $y = 16 - x^2$. Of all such rectangles, what are the dimensions of the one with the greatest area?



$A = \ell \cdot w$

$A(x) = 2x(16 - x^2)$

$A(x) = 32x - 2x^3$

$A'(x) = 32 - 6x^2$

$32 - 6x^2 = 0$

$32 = 6x^2$

$5.\overline{3} = x^2$

$2.31 = x$

$2.31 + 2.31 = 4.62$

$16 - (2.31)^2 = 10.6639$

4.62×10.66

3. Use the linear approximation $(1 + x)^k \approx 1 + kx$ to find an approximation for the function $f(x) = 1/(\sqrt{(4+x)})$ for the values of x near zero.

Tangent Line

$(4+x)^{-1/2}$

$f(x) = \frac{1}{\sqrt{4+x}}$ $\quad x = 0,\ y = \frac{1}{2}$

$f(0) = \frac{1}{\sqrt{4+0}} = \frac{1}{2}$

$f'(x) = -\frac{1}{2}(4+x)^{-3/2} \cdot 1$

$f'(0) = -\frac{1}{2}(4+0)^{-3/2} \cdot 1 = -.0625$

$y - \frac{1}{2} = -.0625(x - 0)$

or

$y = -.0625x + .5$

4. Find the differential, dy, of $y = (x + 1)^4$.

$\frac{dy}{dx} = 4(x+1)^3 \cdot 1$

$dy = 4(x+1)^3 dx$

5. Find the differential, dy, of $y^2 + 2xy = 3$.

u v

$2y + 2x\frac{dy}{dx} + 2y = 0$

$2x\frac{dy}{dx} = -4y$

$\frac{dy}{dx} = \frac{-4y}{2x}$

$\frac{dy}{dx} = -\frac{2y}{x}$

$dy = -\frac{2y}{x} \cdot dx$

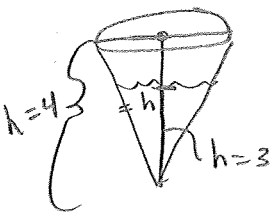
6. Let $y = \sin(2x - 3)^2$. Find dy.

$y = [\sin(2x-3)]^2$

$\frac{dy}{dx} = 2(\sin(2x-3)) \cdot \cos(2x-3) \cdot 2$

$dy = 4\sin(2x-3)\cos(2x-3)dx$

7. A conical paper cup is 3 inches across the top and 4 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep? Express the answer in inches per minute.



$r = 1.5in$

$\frac{dv}{dt} = -2 in^3/min$

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (.375h)^2 h$

$V = \frac{1}{3}\pi (.140625) h^3$

$V = .046875\pi h^3$

$\frac{dv}{dt} = 3(.046875\pi) h^2 \frac{dh}{dt}$

$-2 = .441786(9)\frac{dh}{dt}$

$\frac{dh}{dt} = .503 \ in/min$

$\frac{r}{h} = \frac{1.5}{4}$

$\frac{r}{h} = .375$

$r = .375h$

8. Air is being pumped into a spherical balloon at the rate of 7 cubic centimeters per second. What is the rate of change of the radius at the instant the volume equals $36\prod$?

$\frac{dv}{dt} = 7 cm^3/sec$

$\frac{dr}{dt} = ?$

$V = 36\pi$

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3$

$r = 3$

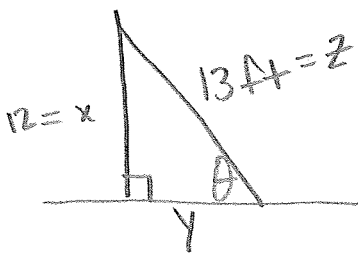
$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$7 = 4\pi (3)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = .06189 \ cm/sec$

9. A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at a constant rate of 2 inches per second, how fast is the angle formed by the ladder and the ground changing (in radians per second) at the instant when the top of the ladder is 12 feet above the ground?



$2in \times \frac{1ft}{12in} = \frac{1}{6}ft$

$\frac{dy}{dt} = 2in/sec = \frac{1}{6}ft/sec$

$\frac{d\theta}{dt} = ?$

$\sin\theta = \frac{12}{13}$

$\theta = 1.176 \ rad$

*Do Not Know $\frac{dy}{dt}$! so do not want to use x!

$\frac{dz}{dt} = 0$

$12^2 + y^2 = 13^2$

$y = 5$

$\cos\theta = \frac{y}{z}$

$-\sin\theta \frac{d\theta}{dt} = \frac{z\frac{dy}{dt} - y\cdot\frac{dz}{dt}}{z^2}$

$-\sin(1.176)\frac{d\theta}{dt} = \frac{13(1/6) - 0}{169}$

$-\sin(1.176)\frac{d\theta}{dt} = .01282$

$\frac{d\theta}{dt} = -.0139$