

Section 4.5 Exercises

1. (a) $f'(x) = 3x^2 - 2$

We have $f(2) = 7$ and $f'(2) = 10$.

$$\begin{aligned} L(x) &= f(2) + f'(2)(x-2) \\ &= 7 + 10(x-2) \\ &= 10x - 13 \end{aligned}$$

(b) Since $f(2.1) = 8.061$ and $L(2.1) = 8$, the approximation differs from the true value in absolute value by less than 10^{-1} .

3. (a) $f'(x) = 1 - x^{-2}$

We have $f(1) = 2$ and $f'(1) = 0$.

$$\begin{aligned} L(x) &= f(1) + f'(1)(x-1) \\ &= 2 + 0(x-1) \\ &= 2 \end{aligned}$$

(b) Since $f(1.1) = 2.009$ and $L(1.1) = 2$, the approximation differs from the true value by less than 10^{-2} .

5. (a) $f'(x) = \sec^2 x$

We have $f(\pi) = 0$ and $f'(\pi) = 1$.

$$\begin{aligned} L(x) &= f(\pi) + f'(\pi)(x-\pi) \\ &= 0 + 1(x-\pi) \\ &= x - \pi \end{aligned}$$

(b) Since $f(\pi + 0.1) \approx 0.10033$ and $L(\pi + 0.1) = 0.1$, the approximation differs from the true value in absolute value by less than 10^{-3} .

9. (a) $f(x) = (1-x)^6 = [1+(-x)]^6 \approx 1 + 6(-x) = 1 - 6x$

(b) $f(x) = \frac{2}{1-x} = 2[1+(-x)]^{-1} \approx 2[1+(-1)(-x)] = 2 + 2x$

(c) $f(x) = (1+x)^{-1/2} \approx 1 + \left(-\frac{1}{2}\right)x = 1 - \frac{x}{2}$

11. $x = 100$

$$f'(100) = \frac{1}{2}(100)^{-1/2} = 0.05$$

$$f(100) = 10 + 0.05(101-100) = 10.05$$

14. $x = 81$

$$f'(81) = \frac{1}{2}(81)^{-1/2} = \frac{1}{18}$$

$$y = 9 + \frac{1}{18}(80-81)$$

$$y = 9 - \frac{1}{18} = 8.9\bar{4}$$

19. (a) Since $\frac{dy}{dx} = 3x^2 - 3$, $dy = (3x^2 - 3)dx$.

(b) At the given values,

$$dy = (3 \cdot 2^2 - 3)(0.05) = 9(0.05) = 0.45.$$

21. (a) Since $\frac{dy}{dx} = (x^2) \left(\frac{1}{x} \right) + (\ln x)(2x) = 2x \ln x + x$,

$$dy = (2x \ln x + x)dx.$$

(b) At the given values,

$$dy = [2(1) \ln(1) + 1](0.01) = 1(0.01) = 0.01$$

25. (a) $y + xy - x = 0$

$$y(1+x) = x$$

$$y = \frac{x}{x+1}$$

$$\text{Since } \frac{dy}{dx} = \frac{(x+1)(1) - (x)(1)}{(x+1)^2} = \frac{1}{(x+1)^2},$$

$$dy = \frac{dx}{(x+1)^2}.$$

(b) At the given values,

$$dy = \frac{0.01}{(0+1)^2} = 0.01.$$

31. (a) $\Delta f = f(0.1) - f(0) = 0.21 - 0 = 0.21$

(b) Since $f'(x) = 2x + 2$, $f'(0) = 2$.

Therefore, $df = 2 dx = 2(0.1) = 0.2$.

(c) $|\Delta f - df| = |0.21 - 0.2| = 0.01$

33. (a) $\Delta f = f(0.55) - f(0.5) = \frac{20}{11} - 2 = -\frac{2}{11}$

(b) Since $f'(x) = -x^{-2}$, $f'(0.5) = -4$.

Therefore, $df = -4 dx = -4(0.05) = -0.2 = -\frac{1}{5}$

(c) $|\Delta f - df| = \left| -\frac{2}{11} + \frac{1}{5} \right| = \frac{1}{55}$

37. Note that $\frac{dV}{dx} = 3x^2$, so $dV = 3x^2 dx$. When x changes from

a to $a + dx$, the change in volume is approximately

$$3a^2 dx.$$